History of Math

Course Materials:

All relevant materials for the course MAT108 (spring 2013) at mc3.edu

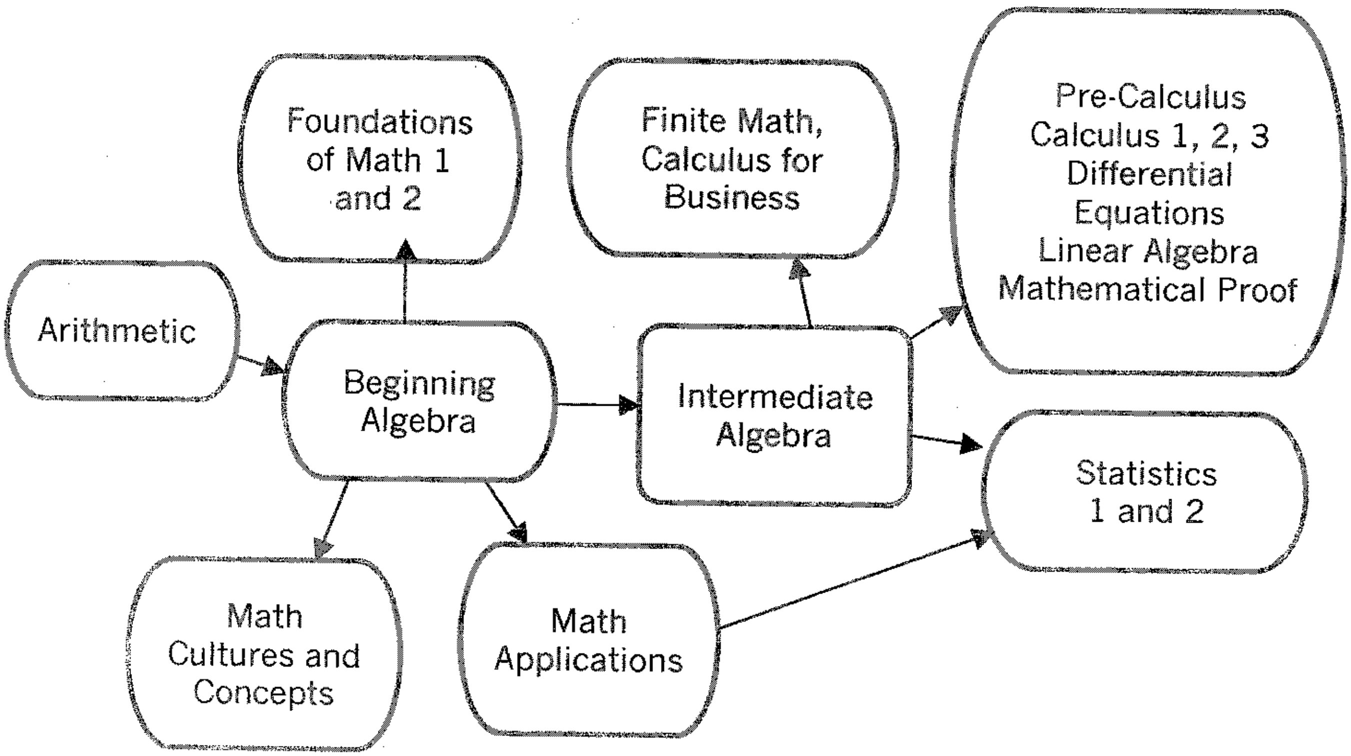
History of Math

Table of Contents:

Why study mathematics?

Mathematics is a window on the universe, and a master key to unlocking its secrets.

"The universe cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language ... without which... it is humanly impossible to comprehend a single word." - Galileo



MAT 010 Fundamentals of Arithmetic
MAT 010B Review of the Fundamentals of Arithmetic
MAT 011 Beginning Algebra
MAT 011B Beginning Algebra W/Review of Arithmetic
MAT 100 Intermediate Algebra
MAT 100B Intermediate Algebra and Review
MAT 103 Foundations of Math
MAT 104 Foundations of Mathematics II
MAT 106 Math Applications
MAT 108 Mathematics Culture and Concept
MAT 125 Discrete Mathematics
MAT 131 Introduction to Stat 1
MAT 132 Introduction to Stat 2

MAT 140 Finite Mathematics for Business
MAT 142 Calculus for Business, Social Science
MAT 161 Precalculus I
MAT 162 Precalculus II
MAT 188 Calculus With a Review of Functions I
MAT 189 Calculus With a Review of Functions II
MAT 190 Calculus w Analytic Geometry I
MAT 201 Calculus w Analytic Geometry II
MAT 202 Calculus w Analytic Geometry III
MAT 211 Foundations of Mathematical Proof
MAT 220 Linear Algebra w/Applications
MAT 223 Differential Equations
MAT 299 Indep Study / Mathematics
MAT M99 Topics in Mathematics

Montgomery County Community College
For more information: www.mc3.edu
Central: (215) 641-6577
West: (610) 718-1906

Section Information

Title	Mathematics Culture & Concept
Course Section Number	MAT*108*KC
Description	A course designed primarily for liberal arts students, which shows how mathematics has developed concomitantly with civilization. The applications demonstrate that mathematics is related not only to the physical sciences but also to the social sciences, to philosophy, logic, religion, literature, and the arts. This course does not satisfy the MAT 100 prerequisite requirement for MAT 125, MAT 131, MAT 140, or MAT 161. Prerequisite: Math placement test recommendation "UND 116" (under MAT 116) or "ABV 100" (above MAT 100) or MAT 011, with a minimum grade of "C."

Credits 3.00 CEUs

Start Date 23 January 2013 End Date 15 May 2013

Academic Level	UG - Undergraduate

Meeting Information

01/24/2013-05/14/2013 Lecture Tuesday, Thursday 12:45PM - 02:10PM, Parkhouse Hall, Room 128

Faculty name	Phone	Extension	E-mail address	Instructional Method
Jeffrey Zilahy				Lecture

Prerequisites

MAT011/011B

Supplies

None

Title	Author	Publisher	Copyright	ISBN	Price	Req.	Comments
Textbook Information From Barnes & Noble							

Additional Comments

MONTGOMERY COUNTY COMMUNITY COLLEGE
SYLLABUS SPRING 2013
MAT 108: MATHEMATICS CULTURE & CONCEPTS
(MAT-108-KC)
3-3-0

COURSE DESCRIPTION:
A course, designed primarily for liberal arts students, which shows how mathematics has developed concomitantly with civilization. The applications demonstrate that mathematics is related not only to the physical sciences but also to the social sciences, to philosophy, logic, religion, literature and the arts. This course does not satisfy the MAT 100 prerequisite requirement for MAT 125, MAT 131, MAT 140 or MAT 161.

PREREQUISITE(S):
Students must have successfully completed or tested out of:
Math placement test recommendation "UND 116" (under MAT 116) or "ABV 100" (above MAT 100) or MAT 011, with a minimum grade of "C"

MEETING INFORMATION:
Parkhouse Hall
Room 128
TTH
12:45PM – 2:10PM

INSTRUCTOR INFORMATION:
Jeffrey Zilahy (pronounced Zil-ahh-he)
EMAIL: jzilahy@mc3.edu
NOTE: All official email will be handled via this MC3 email and Blackboard.
OFFICE: Adjunct Office 99 (Parkhouse)

COURSE MATERIALS *(recommended):*
1. Textbook: Berlinghoff, William and Gouvea, Fernando (2004). *Math through the Ages. Oxton House Publishers*
2. CALCULATOR: You will need a calculator for this course, TI-83 or TI-84 is ideal.
3. ADDITIONAL: Some sort of pen(cil), a notebook and most importantly a good attitude. Other learning materials may be required and made available directly to the student and/or via the College's Libraries and/or course management system.

GRADING:

CONTENT	% of Final Grade	NOTES
HW, QUIZZES	30	Response papers, any HW, quizzes
RESEARCH PAPER	20	At least 3 pages, topic TBD
MIDTERM	25	About halfway through course
FINAL	25	Cumulative

LEARNING OUTCOMES:

This is a general overview, as we can expect to cover more than this list entails.

1. Trace the historical development of mathematics from the beginning of civilization to present day.

2. Show how real world problems created a need for the development of mathematics, and how mathematics solved those quandaries.

3. Learn about some of the key mathematicians throughout history and the contributions they made. This includes, but is not limited to: Archimedes, Descartes, Euclid, Gauss, Euler, Leibniz, Newton, Pythagoras and Erdos. This will also include a focus on female mathematicians, who often were relegated as they made their own key contributions.

4. The Development of Geometry, from Euclidean to coordinate geometry to modern day fractal interpretations.

5. The development of numbers including Zero, Negative and Irrational Numbers.

6. The birth and development of the Calculus.

7. Modern day mathematics, with a focus on the discrete side, and its implications on the digital revolution.

GENERAL POLICIES & PERSPECTIVES:

How to succeed in this course: Math is not unlike anything else in this world, namely, practice makes perfect. The more you put into something, the more you will get out of it. In addition, frustration is a completely normal emotion in learning mathematics, what separates the successful students from the not-so-successful students is how they internalize this frustration. Therefore, it is helpful to become very cognizant of the internal monologue you have with yourself as you attempt problems, and be sure to keep the negativity away. It is a bit cliché, but the fact is that attendance is key, if you were a professional football player would you skip practice? Why not? Well, the same goes for learning mathematics. Being a successful student is less about having some magical mathematical skills and more about just developing good habits. For example don't get behind in the material. For some that might be a few minutes extra a week here or there, and for others it may be hours and hours of extra work. The key here is to not worry about where you perceive your math abilities to be, remember, no matter who you are, there are always some people who know more than you and some people that will know less than you.

Academic Honesty: Cheating is obviously completely unacceptable, not to mention really lame. Earn what you earn, and don't undermine your own intelligence by trying to take shortcuts. There are no shortcuts in math, the best way to succeed is simply to work hard, ask questions and most importantly never give up.

Work: Show work! Due to the precise nature of mathematics, we are all likely to make little mistakes along the way. The more work you show the less chance you have of making little mistakes and the easier it is for me to give partial credit. Get in the habit of writing down what you know or what the problem tells you, and drawing diagrams or pictures whenever possible.

ADDITIONAL INFORMATION:

CLASSROOM ETIQUETTE

I expect you to act like adults in a learning environment. Remember, when you cause disruptions, *you are hindering others abilities to learn, not just your own.* You are expected to follow all rules in the student's code of conduct in this classroom. For more information on the Student Code of Conduct please the website: http://www.mc3.edu/aboutus/policies/sa-4/conduct.aspx

STUDENT SUPPORT REFERRAL TEAM / SSRT.

The SSRT is aware that students face many challenges in and out of the classroom. The SSRT is a free, confidential referral service available to all students that works to connect students with College and community resources, and caring professionals. It involves a support team of counselors, faculty, and staff who assist students dealing with such issues as emotional distress, anxiety, eating disorders, abuse, depression, grief, potential violence, and substance abuse. If students recognize that they have concerns, they may contact the SSRT directly for assistance, either by sending an email to a secure, confidential address (StudentReferral@mc3.edu), or by visiting the Student Success Center located in College Hall.

TUTORIAL SERVICES

Please visit the Foundational Skills Lab or take advantage of the Live Online Tutoring! The lab offers **FREE tutoring** and is a very useful resource. FREE subject-area tutoring, academic workshops, and study skills specialists are available at Blue Bell Campus's Foundation Skills Lab in College Hall 168 (across from the Cafeteria.) The Foundation Skills Lab helps students develop learning strategies based on their unique learning styles with the goal of creating successful students and independent learners. Contact 215-641-1150, fsl@mc3.edu or via the portal
http://mymccc.mc3.edu/allcampusresources/academicaffairs/dsl/Pages/default.aspx

Live Online Tutoring
Tutorial Services offers FREE live online tutoring.
Students have two options for online tutoring:

- Discussion board based tutoring via Blackboard gives students the ability to post their questions to a tutor twenty four hours a day. The tutor will respond to the post within twenty four hours.

- Live after hours online tutoring gives students the ability to chat with a tutor through a web based conferencing tool. These are late night/early morning hours.

Visit Tutorial Services for more information on Live Online Tutoring and the available times for this semester.
http://mymccc.mc3.edu/allcampusresources/academicaffairs/lal/Pages/default.aspx

IMPORTANT DATES
Classes Begin --- January 24, 2013
Spring Break --- No Classes --- March 18 – March 24, 2013
Classes End --- Tuesday, May 7, 2013
Reading Day --- Wednesday, May 8, 2013
Final Exams --- May 9 – May 15, 2013

Students With Disabilities:
Students with disabilities may be eligible for accommodations in this course. Please contact the Disability Services Center in College Hall 225 for more information at (215) 641-6575 or disabilities@mc3.edu . Students at West Campus should contact the Student Success Center in South Hall or call (610) 718-1853.
Or visit the Students with Disabilities website:
http://www.mc3.edu/campusLife/student-resources/disabilities/

Veterans:
Student veterans may eligible for benefits and services related to military service. At Central Campus, contact the Military and Veteran Affairs Advisor at (215) 619-7307 in CH 209 (inside the student success center) to learn about educations benefits and healthcare entitlements. At the West Campus, contact Michael Ondo in South Hall 151 or at (610) 718-1857 for the veterans' resources.
Or visit the Veterans website:
http://www.mc3.edu/campusLife/student-resources/veterans

Class Cancellation Policy:
Radio Codes 320 for day and 2320 for evening.
Please sign up for the texting service offered by the college. If you sign up for this service, you will be notified via text message to college closings and delays.
Please call the MCCC Main Number 215-641-6300, in case of inclement weather or other emergency that may cause a class to be cancelled.
Please check Blackboard, if I need to cancel class you be notified via Blackboard announcement. or check the syllabus on canceling classes and notification.

Early Alert:
As part of our commitment to promote the academic success of all our students, the College utilizes an Early Alert system. Faculty may alert Academic Advisors about issues related to student absence and/or academic difficulty, if they are not resolved through discussion between the faculty and student. Please respond promptly to any Early Alert notification you receive

through email, phone or mail, as your Academic Advisor hopes to offer you assistance in resolving these issues.

WITHDRAWAL
The last day to withdraw from the course is May 15th, 2013.
Note: The last day to withdraw without your instructor's signature is March 19th, 2013.
Note: You must attend at least 60% of the classes to be permitted to withdraw.

AUDIT
No audits are permitted in this class.

Lesson 1 Summary: January 24th
Introduction to Course
Pervasiveness and the beginning of math
Civilizations Early Number Systems

Lesson Handouts
As I was going to St. Ives (Wikipedia)
4 pages on Ancient Number Systems (MAT010)

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty -- a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."

Bertrand Russell (1872-1970), The Study of Mathematics

Welcome to Mathematics Culture and Concepts MAT108

Prepare to take a journey through history and learn about how math has shaped everything we take for granted in our lives and really is the closest thing to a cosmic language humanity has. The story of mathematics is an exciting one, believe it or not!

Today's Agenda:

1. Go over Syllabus
2. Thoughts on math
3. Riddle Fun
4. Introduction to Antique Number systems.

http://www.youtube.com/watch?feature=player_embedded&v=3uYBoWH3nFk

Why do people hate math?

We will watch this quick funny/silly montage and then:
Take several minutes and on a sheet of paper to hand in, write about why our culture seems to have such a love/hate relationship with math. Make sure you discuss your own thoughts on math. Keep in mind that math is responsible for **your cell phone**, the intrawebs, PS3, the chair you are sitting in, etc, etc, etc...that said, there are no wrong answers here, just provide depth of opinion.

"As I was going to St Ives" is a traditional English language nursery rhyme which is generally thought to be a riddle. (c. 1730)

As I was going to St Ives
I met a man with seven wives
Every wife had seven sacks
Every sack had seven cats
Every cat had seven kits
Kits, cats, wives.
How many were going to St Ives?

"ONE" ANSWER:
We want to know the number of living things in the group the man meets.This is represented by:
1 man + 7 wives + 343 cats (7cats*7sacks*7wives) + 2401 kits (343 cats*7) = 1 + 7 + 343 + 2401 = **2752**

1,1,2753,63,1,1,2331,0,2751,2753,

THE BEGINNING....

Mathematics started alongside the same time that the written word was born. Math is inherent in all humans, we do it without even realizing. For example, when you hit a baseball, or sit in a chair, you are actually doing advanced spatial geometry to ensure your action is a success! Even animals count, birds and fish use the position of stars to navigate and ants have built in pedometers.

As humans started forming civilization, mathematics was an absolute necessity to ensure that we could keep track and stay organized. As civilization evolved and grew, so too did mathematics. It can be argued that the level of civilization a society is at (iphone 5 vs. tin cups and a string) is directly linked to how much mathematics it knows.

It is not known for sure exactly how, when and who really started mathematics.

However, we have some clear historical evidence of some of the earliest known mathematical artifacts. For example, the **ishango bone** is one of oldest known math objects, it is estimated to be over 20,000 years! That is over 250 lifetimes in a row. It is the bone of a baboon, and has etchings in it which correspond to numbers, even today, there is dispute over the exact use and interpretation of the numbers of the bone.

The first real body of mathematical knowledge started in Mesopotamia and the ancient Egyptian societies. There was also mathematical development in Indian and Chinese societies, although the depth of knowledge is less clear.

Today, we will explore some of the number systems of these ancient societies.

Before we dive in, consider a simple question.

How many digits do we modern humans use to construct any number?

Why do you think we use this particular number of digits?

Do we have to use this number to count?

What is binary?

345 =

#	BABYLONIAN	GREEK	ROMAN	MAYAN
0				
1				
5				
10				
42				

1.1 History of Numbers

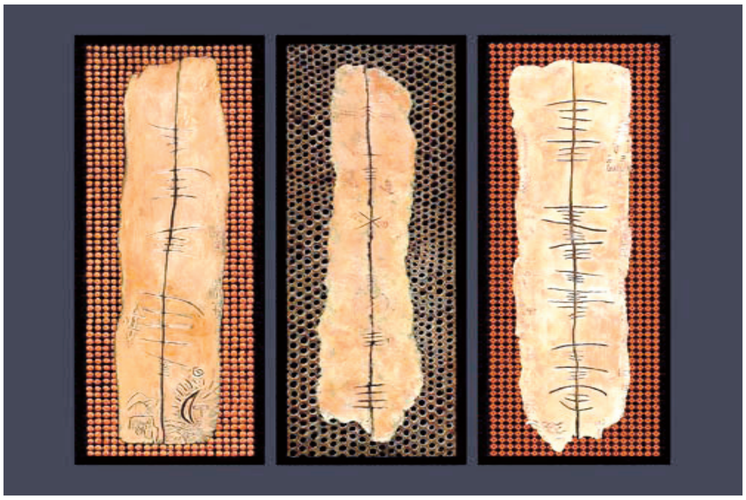
In this unit, we will look at ancient civilizations and how they used numbers and operations. We will see how our present number system evolved from the numeration systems of many cultures. It is through an understanding of the past that we can appreciate the present.

Learning Outcomes:
- Demonstrate an understanding of the evolution of our numeration system by relating counting, place value, and place holder concepts from the past to the present
- Recognize early numeration systems
- Write numbers in Egyptian, Babylonian, Greek and Roman
- Demonstrate a knowledge of our present system of place values and expanded notation

In the beginning…

I. Tally marks using "grouping" were the first known form of counting. Larger grooves and crossed slashes were used to represent and record larger quantities. These marks can be found dating back to 3000 B.C. in Egypt and Babylonia.

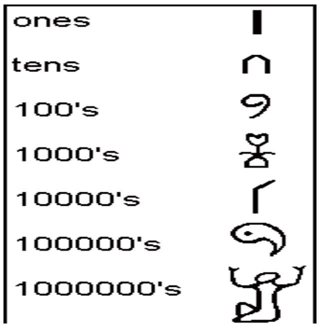
The image below, "Three" by Marion Drennen, depicts early tally sticks on a clay tablet.



"Three" by Marion Drennen, St. Francisville, Louisiana, http://quantumconnectionsart.blogspot.com
Represented by Brunner Gallery, Covington, Louisiana, www.brunnergallery.com.

II. The **Egyptians** used symbols painted in their pottery, cut numbers in stone and scrolled on papyrus to express counting. We have evidence that this system dates back to 3400 B.C.

The following chart depicts the Egyptian symbols for numbers:



These *hieroglyphic* symbols are a stroke, arch, rope, flower, finger, tadpole and an astonished man. These are pictorial symbols from everyday life. To make writing easier to read the repeated symbols can be grouped in two, three or four symbols arranged vertically.

The number 672 would be ٩٩٩٩٩٩ПППППППII or ٩٩٩ППППII
٩٩٩ППП

- The Egyptian numeration system was *additive* and used *grouping by tens*.
- The system was very cumbersome when writing large numbers and when operations were performed.

Write the following numbers using the Egyptian numeration system:

a. 345

b. 42,320

c. 567,922

d. What are the advantages and disadvantages in using the Egyptian system? Explain.

III. The **Greeks** used letters of their alphabet for their numbers. It was a *ciphered* numeration system that dates back to 3000 B.C.

The following chart depicts the Greek system for numbers:

1 = α	10 = ι	100 = ρ
2 = β	20 = κ	200 = σ
3 = γ	30 = λ	300 = τ
4 = δ	40 = μ	400 = υ
5 = ε	50 = ν	500 = φ
6 = ς (ϝ)	60 = ξ	600 = χ
7 = ζ	70 = ο	700 = ψ
8 = η	80 = π	800 = ω
9 = θ	90 = ϙ	900 = ϡ

EXAMPLE:

The number 672 would be χοβ .

Write the following numbers using the Greek numeration system:

a. 345

b. 949

c. 888

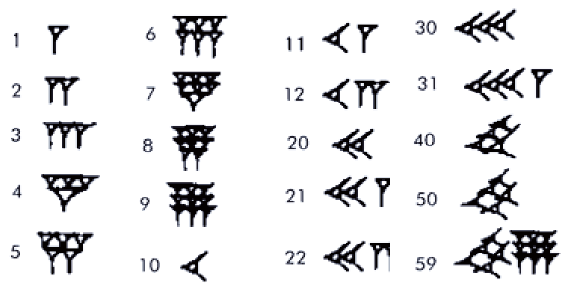
d. Convert to our number system: Φμβ

e. What are the advantages and disadvantages in using the Greek system? Explain.

The Greeks were the first to use a comma to write large numbers.
Example: **,β** = 2000 or (2 x1000)

IV. The **Babylonian** numeration system is one of the oldest. It dates back to 5000 B.C. Babylonians began using tally marks, which evolved into a wedge shaped symbol called a *cuneiform*. Many of their clay tablets are still around today. The Babylonians used a base 60 system instead of the base 10 system that we use today. It is believed to be related to the concept of time – 24 hours in a day, 60 minutes in an hour and 60 seconds in a minute. There are only two symbols, a modernized wedge pointed down, ▼, for one and a modernized wedge pointing to the left for ten, **< .**

The following chart depicts the Babylonian system for numbers:



EXAMPLES:

- The number 42 would be **<<<<▼▼**
- The number 672 would be **<▼ <▼▼**

↓ ↓

(11x 60) + (12 x 1)

672 = (11 groups of 60) + (12 groups of 1)

> In approximately 3400 B.C. a space was used to indicate a place holder (zero was only an idea; the Babylonians knew they needed something to distinguish between place values). A space was used to show breaks between place values.

Write the following numbers using the Babylonian numeration system:

a. 48

b. 67

c. 132

d. 3702

e. What are the advantages and disadvantages in using the Babylonian system? Explain.

As I was going to St Ives

"**As I was going to St Ives**" Roud #19772	
Written by	Traditional
Published	c. 1730
Written	England
Language	English
Form	Nursery rhyme/riddle

"**As I was going to St Ives**" is a traditional English language nursery rhyme which is generally thought to be a riddle. It has a Roud Folk Song Index number of 19772.

Lyrics

The most common modern version is:

As I was going to St Ives
I met a man with seven wives
Every wife had seven sacks
Every sack had seven cats
Every cat had seven kits
Kits, cats, sacks, wives.
How many were going to St Ives?[1]

Origins

The earliest known published version of it comes from a manuscript dated to around 1730 (but it differs in referring to "nine" rather than "seven" wives).[1] The modern form was first printed around 1825.[1] A similar problem appears in the Rhind Mathematical Papyrus (Problem 79), dated to around 1650 BC:

There are seven houses;
In each house there are seven cats;
Each cat catches seven mice;
Each mouse would have eaten seven ears of corn;
If sown, each ear of corn would have produced seven hekat of grain.
How many things are mentioned altogether?[2]

There are a number of places called St Ives in England and elsewhere. It is generally thought that the rhyme refers to St Ives, Cornwall, when it was a busy fishing port and had many cats to stop the rats and mice destroying the fishing gear, although some people argue it was St Ives, Huntingdonshire as this is an ancient market town and therefore an equally plausible destination.[3][4]

Answers

All potential answers to this riddle are based on its ambiguity because the riddle only tells us the group has been "met" on the journey to St. Ives and gives no further information about its intentions, only those of the narrator. As such, any one of the following answers is plausible, depending on the intention of the other party:

- **1**: If the group that the narrator meets is assumed not to be travelling to St. Ives (this is the most common assumption),[1] the answer would be *one* person going to St. Ives; the narrator.
- **2802**: If the narrator met the group as they were also travelling to St. Ives (and were overtaken by the narrator, plausible given the large size of the party),[1] the answer in this case is *all* are going to St. Ives; see below for the mathematical answer.
- **2800**: If the narrator and the group were all travelling to St. Ives, the answer could also be *all except the narrator and the man* since the question is ambiguous about whether it is asking for the total number of entities travelling or just the number of kits, cats, sacks and wives. This would give an answer of 2,800 — 2 fewer than the answer above.
- **2**: Two is also a plausible answer. This would involve the narrator meeting the man who is assumed to be travelling to St. Ives also, but plays on a grammatical uncertainty, since the riddle states only that the man has seven wives (and so forth), but does not explicitly mention whether the man is actually accompanied by his wives, sacks, cats, and kittens.
- **0**: Yet another plausible is *zero*, once again playing on a grammatical uncertainty. The last line of the riddle states "kits, cats, sacks, wives ... were going to St. Ives?" Although the narrator clearly states he is going to St. Ives, by definition he is not one of the kits, cats, sacks, or wives, and based on the common assumption that the party was not going to St. Ives, the answer is zero.
- **2753**: The sacks are not a person or animal and therefore cannot be in the calculation. It was not the number of things, but of "persons" the narrator met. 49 adult cats 343 kittens per wife of whom he had seven (7 × 392) = 2744 plus the seven wives 2751 plus the man + the narrator → 2753 persons and animals.
- **9**: There are nine people involved, who may be going to St. Ives. The animals are all in the sacks, so they, as well as the sacks themselves, are "being taken", rather than "going".
- **7**: There are nine people involved, who are the only ones who may be going to St. Ives, all the others "being taken" there. But since the question is limited to "Kits, cats, sacks, wives", this excludes the man and the narrator, leaving seven.

Rhind mathematical papyrus

A similar problem is found in the Rhind Mathematical Papyrus (Problem 79), dated to around 1650 BC. The papyrus is translated as follows:[5]

A house inventory:

		houses	7
1	2,801	cats	49
2	5,602	mice	343
4	11,204	spelt	2,301 [sic]
		hekat	16,807
Total	19,607	Total	19,607

The problem appears to be an illustration of an algorithm for multiplying numbers. The sequence 7, 7 × 7, 7 × 7 × 7, ..., appears in the right-hand column, and the terms 2,801, 2 × 2,801, 4 × 2,801 appear in the left; the sum on the left is 7 × 2,801 = 19,607, the same as the sum of the terms on the right. Note that the author of the papyrus listed a wrong value for the fourth power of 7; it should be 2,401, not 2,301. However, the sum of the powers (19,607) is correct.

The problem has been paraphrased by modern commentators as a story problem involving houses, cats, mice, and grain, although in the Rhind Mathematical Papyrus there is no discussion beyond the bare outline stated above. The hekat was $^1/_{30}$ of a cubic cubit (approximately 4.8 l, 1.1 imp gal; 1.3 US gal).

Use in popular culture

- In the 1995 film *Die Hard with a Vengeance*, the rhyme is presented to the protagonists by the villain as a riddle, giving them thirty seconds to telephone him on the number "555 plus the answer" or a bomb would detonate. After several guesses, they eventually solve the riddle, calling the number 555-0001 which proves to be correct. They missed the 30-second deadline, but the bomb did not explode since the villain had not said "Simon says."
- The rhyme was recited by Mary Murphy's character while caring for a cat with seven kittens in the movie *A Man Alone*. Later the character played by Ray Milland who overheard the rhyme offers her the answer and Murphy's character explains that she alone was going to St. Ives.
- The rhyme was also the basis of a *Sesame Street* Muppet skit from the show's first season, in which the boy Muppet holding a numeral 7 sings the rhyme as a song to the girl Muppet twice (the second time, the girl is busy writing down the calculations) and finally, in keeping true to the spirit of the riddle, reveals the answer as 1 (the traditional answer), because *he* was going to St. Ives and the kits, cats, sacks and wives were going the other way. Then the girl turns the tables on the boy and asks how many were going the other way. She then reveals the mathematical answer from her calculations: 1 man + 7 wives + 49 sacks + 343 cats + 2,401 kittens, which comes to 2,801. Astonished, the boy responds, "How about that?!"
- The rhyme was also featured in a *Pogo* comic story, "More Mother Goosery Rinds" in which Albert Alligator himself portrays Mother Goose and Pogo a traveling musician. After going over several Mother Goose rhymes they get to the St. Ives riddle, albeit replacing "seven" with "forty" and while Albert (Mother Goose) keeps trying to cogitate the answer, Pogo boasts he knows it...and he answers "one", which baffles "Mother Goose" (Albert Alligator). Pogo says that if the kits, cats, sacks and wives weren't going to St. Ives, maybe they were going somewhere else, such as Altoona, Pennsylvania. So Albert again recites the riddle, this time ending with "Kits, cats, sacks, wives, how many were going to...ALTOONA??" But by this time, Pogo has already gone upon his way.
- Mad magazine used it in at least two articles over the years for the following parodies:

  As I was going to St Ives

  I met a man with seven wives

  Of course, the seven wives weren't his

  But here in France, that's how it is

and

  As I was going to St Ives

  I met a man with seven wives

  I know this sounds absurd and loony

  But that poor man was Mickey Rooney!

- British poet and humorist Colin West wrote a satire on "As I was going to St Ives", called "As I Went Down To Milton Keynes". The items listed are "a king with seven queens", and for every queen a prince, for every prince a princess, for every princess an earl, for every earl a lady, for every lady a baby, and for every baby a cat.

Notes

[1] I. Opie and P. Opie, *The Oxford Dictionary of Nursery Rhymes* (Oxford University Press, 1951, 2nd edn., 1997), pp. 376-7.

[2] "Transcript EPISODE 17 - RHIND MATHEMATICAL PAPYRUS" (http://www.bbc.co.uk/ahistoryoftheworld/about/transcripts/episode17/). *A history of the world*. BBC. . Retrieved 26 February 2012.

[3] Hudson, Noel (1989), *St Ives, Slepe by the Ouse*, St Ives Town Council, p. 131, ISBN 978-0-9515298-0-5

[4] Flanagan, Bridget (2003), *The St Ives Problem, a 4000 Year Old Nursery Rhyme?*, ISBN 0-9540824-1-9

[5] Maor, Eli (2002) [1988], "Recreational Mathematics in Ancient Egypt" (http://pup.princeton.edu/books/maor/sidebar_a.pdf), *Trigonometric Delights*, Princeton University Press, pp. 11–14 (in PDF, 1–4), ISBN 978-0-691-09541-7, , retrieved 2009-04-19

References

- Oystein Ore, "Number Theory and its History", McGraw- Hill Book Co, 1944

Lesson 2 Summary: January 29th
2/n table
(Unit) Fraction Skills

Lesson Handouts
Dung Beetle Article (BBC)
America & Fractions Article (Scientific American)

The essence of mathematics is not to make simple things complicated, but to make complicated things simple.
~S. Gudder

Jan 28-9:44 AM

WARM UP:

1. What is one of the oldest historical mathematical artifacts ever found?

2. Give at least one example of how animals use (or may use) math.

3. Of the antique number systems we covered last week, which is your favorite and why?

4. How many digits do we use to make numbers? What about computers?

Jan 28-9:41 AM

You all got 10 out of 10 on your thoughts on math. Here are a few of your highlights:

"I used to hate math, now I like math and have come to be rather good at it."

"Math at its surface is very unrelatable."

"Math is life!"

"Disliking math can come from struggle."

"People take math for granted."

"I am not a fan of math."

Jan 29-9:47 AM

FUN MATH
"The Birthday Paradox"

How many possible birthdays are there?

What is the chance that you and your neighbor have the same birthday?

What if we had 23 random people pulled off the street? What is the chance that we find a matching birthday?

Jan 29-11:26 AM

Last week, we started talking about the origins of math. We learned that the Babylonians, Egyptians, Mayans and Romans were some of the earliest civilizations to start to formalize mathematics.

We also learned that even animals do math. Consider this recent article which indicates that even **insects** are using mathematics, in the form of celestial navigation!

Jan 29-10:26 AM

Math is the underlying **software** that runs the **hardware** that we fondly call the Earth/Universe.

Jan 29-10:50 AM

Today, let's dive deeper into the Egyptian civilization and some of their mathematics.

"Accurate reckoning for inquiring into things, and the knowledge of all things, mysteries...all secrets."

This is in the introduction to one of the most famous mathematical artifacts we have ever uncovered...the **Rhind Papyrus**.

Jan 29-10:34 AM

Here is a image of part of the Rhind Papyrus...

Jan 29-10:38 AM

The Rhind Papyrus has many problems dealing with arithmetic, algebra, and geometry.
Consider after all that some of the most dramatic and awe inspiring architecture ever constructed by humans are the Great Pyramids. It is mathematics that allowed these structures to be built and for many centuries, they were the biggest structures on planet earth, nothing to sneeze at!

Jan 29-10:39 AM

Today, we want to consider one particular section of the Rhind Payprus, that dealing with what is known as the **2/n table**.

This table illustrates how the Egyptians dealt with parts of things, namely fractions. Before we do though, we need to review our own knowledge and make sure we are on solid footing in working with fractions.

Before we do that, let's first consider this recent article on America and fractions!

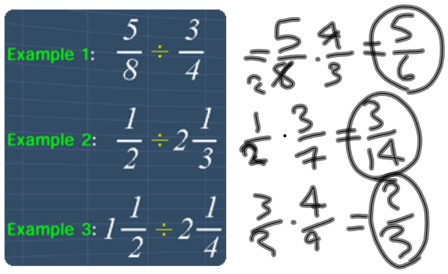
Jan 29-10:44 AM

Perhaps the reason why people are intimidated by fractions is that they are messy! As humans, we like order and the wholeness of things. If you disagree, just consider why we celebrate birthday's like our 30th or 50th with greater frequency than our 34th or 26th.

However, there is simply no good reason why anyone should not be able to comfortably work with fractions, whether it be in the context of multiplication, division, addition or subtraction.

Jan 29-10:52 AM

Does this look tough? Can you do these problems right now? Take a few minutes and try to answer these problems.



Jan 29-10:59 AM

$\div$

$$\frac{\frac{6}{7}}{\frac{2}{5}} = \frac{6}{7}\cdot\frac{5}{2} = \frac{30}{14} =$$

$$\frac{15}{7} \qquad 2\frac{1}{7}$$

Jan 29-1:36 PM

$$\frac{4}{7}\cdot\frac{2}{9} = \frac{8}{63}$$

$$\frac{2}{5}\cdot 6\frac{1}{4} = \frac{2}{5}\cdot\frac{25}{4} = \frac{50}{20}$$

Jan 29-1:23 PM

$$\frac{4}{7} \div \frac{2}{9} =$$

$$\frac{4}{7}\cdot\frac{9}{2} = \frac{18}{7}$$

Jan 29-1:29 PM

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

$$\frac{1}{4} - \frac{3}{4} = \frac{-1}{4}$$

Jan 29-1:40 PM

$$\frac{1}{3}+\frac{2}{5}=\frac{5+6}{15}=$$

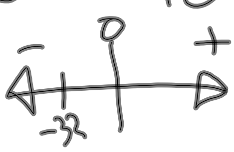
$$\frac{11}{15}$$

Jan 29-1:42 PM

$$\frac{6}{1}-\frac{1}{6}=\frac{36}{6}-\frac{1}{6}$$

Jan 29-1:45 PM

$$\frac{-4}{5}-\frac{7}{8}=\frac{-32\mp 35}{40}=\frac{-67}{40}$$



$$-\frac{-1}{-3}$$

Jan 29-1:51 PM

Now that we are comfortable with dealing with fractions, let's return to the Egyptians and the Rhind Papyrus.

In particular, let's consider the 2/n table found on the Rhind Papyrus. See, the Egyptians were well aware of fractions and they could work with them. **However, they struggled with fractions that were not unit fractions**. What are unit fractions again? Just fractions where the numerator is a 1. 1/3, 1/50 and 1/100 are just a few examples of unit fractions.

Jan 29-11:24 AM

So where we in modern times have no trouble dealing with fractions like 2/7 and 2/13, the Egyptians could only understand those fractions as **the SUM of Unit Fractions**.

Hence, the 2/n table is a list of fractions with numerator two expressed as the decomposition of unit fractions.

So for n =3, we have 2/3. This is just equal to: 1/2 + 1/6 since 3/6 + 1/6 = 4/6 which is 2/3.

Jan 29-11:36 AM

A few more examples:

3/4 = 1/2 + 1/4

6/7 = 1/2 + 1/3 + 1/42

The Egyptians are so well associated with this approach that a fraction written as a sum of distinct unit fractions is called an **Egyptian Fraction**.

http://en.wikipedia.org/wiki/Rhind_Mathematical_Papyrus_2/n_table

http://www.youtube.com/watch?v=WCyXl464pJc

Jan 29-11:48 AM

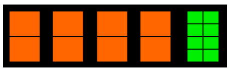
Egyptian Fractions can still be useful, even today!

Consider the following problem:

Suppose Fatima has 5 loaves of bread to share among the 8 workers who have helped dig her fields this week and clear the irrigation channels. How would she break it up evenly?

First Fatima sees that they all get at least half a loaf, so she gives all 8 of them half a loaf each, with one whole loaf left. Now it is easy to divide one loaf into 8, so they get an extra eighth of a loaf each and all the loaves are divided equally between the 5 workers.

On the picture here



they each receive one red part (1/2 a loaf) and one green part (1/8 of a loaf): 4 loaves split into halves and 1 split into eighths

and 5/8 = 1/2 + 1/8.

Jan 29-12:04 PM

Another way in which Egyptian fractions are useful is **comparing fractions**.

Which is larger: 3/4 or 4/5?

Now, we could convert these to decimals, but if we don't have a calculator handy, it might not be so obvious. Egyptian fractions can help us.

Using Egyptian fractions we write each as a sum of unit fractions:

3/4 = 1/2 + 1/4
4/5 = 1/2 + 3/10 and, expanding 3/10 as 1/4 + 1/20 we have
4/5 = 1/2 + 1/4 + 1/20

Therefore, 4/5 is bigger than 3/4 by 1/20!

Jan 29-12:26 PM

Extra Credit for Thursday:

1/x + 1/y + 1/z = ?

$\frac{1}{20} + \frac{1}{17} + \frac{1}{2}$

Jan 29-11:15 AM

#	BABYLONIAN	GREEK	ROMAN	MAYAN
0				
1				
5				
10				
42				

Jan 24-12:10 PM

BBC NEWS

SCIENCE & ENVIRONMENT

24 January 2013 Last updated at 12:34 ET

Dung beetles guided by Milky Way

By Jonathan Amos
Science correspondent, BBC News

They may be down in the dirt but it seems dung beetles also have their eyes on the stars.

Scientists have shown how the insects will use the Milky Way to orientate themselves as they roll their balls of muck along the ground.

Humans, birds and seals are all known to navigate by the stars. But this could be the first example of an insect doing so.

The study by Marie Dacke is **reported in the journal Current Biology**.

"The dung beetles are not necessarily rolling with the Milky Way or 90 degrees to it; they can go at any angle to this band of light in the sky. They use it as a reference," the Lund University, Sweden, researcher told BBC News.

Dung beetles like to run in straight lines. When they find a pile of droppings, they shape a small ball and start pushing it away to a safe distance where they can eat it, usually underground.

Getting a good bearing is important because unless the insect rolls a direct course, it risks turning back towards the dung pile where another beetle will almost certainly try to steal its prized ball.

Dr Dacke had previously shown that dung beetles were able to keep a straight line by taking cues from the Sun, the Moon, and even the pattern of polarised light formed around these light sources.

But it was the animals' capacity to maintain course even on clear Moonless nights that intrigued the researcher.

So the native South African took the insects (*Scarabaeus satyrus*) into the Johannesburg planetarium where she could control the type of star fields a beetle might see overhead.

Importantly, she put the beetles in a container with blackened walls to be sure the animals were not using information from landmarks on the horizon, which in the wild might be trees, for example.

The beetles performed best when confronted with a perfect starry sky projected on to the planetarium dome, but coped just as well when shown only the diffuse bar of light that is the plane of our Milky Way Galaxy.

Dr Dacke thinks it is the bar more than the points of light that is important.

"These beetles have compound eyes," she told the BBC. "It's known that crabs, which also have compound eyes, can see a few of the brightest stars in the sky. Maybe the beetles can do this as well, but we don't know that yet; it's something we're looking at. However, when we show them just the bright stars in the sky, they get lost. So it's not them that the beetles are using to orientate themselves."

And indeed, in the field, Dr Dacke has seen beetles run in to trouble when the Milky Way briefly lies flat on the horizon at particular times of the year.

The question is how many other animals might use similar night-time navigation.

It has been suggested some frogs and even spiders are using stars for orientation. The Lund researcher is sure there will be many more creatures out there doing it; scientists just need to go look.

"I think night-flying moths and night-flying locusts could benefit from using a star compass similar to the one that the dung beetles are using," she said.

But for the time being, Dr Dacke is concentrating on the dung beetle. She is investigating the strange dance the creature does on top of its ball of muck. The hypothesis is that this behaviour marks the moment the beetle takes its bearings.

Jonathan.Amos-INTERNET@bbc.co.uk and follow me on Twitter: **@BBCAmos**

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SCIENTIFIC AMERICAN™

June 16, 2012

America Needs to Study Fractions

Recent research finds that a solid grade school knowledge of fractions and long-form division accurately predicts later success in high school math. Christie Nicholson reports

What part of math was most intimidating when you were in grade school? Maybe it was fractions. Or even worse, long-form division. Somehow splitting numbers really seemed complicated.

And the U.S. might be paying for kids' inability to overcome those early challenges: a new study finds that Americans are falling significantly behind in math aptitude compared with China, Finland, the Netherlands and Canada. And the root cause is deficiencies in knowledge of fractions and division.

Nearly 600 children were tested once when they were 10 to 12 years old and again five years later. Researchers found that a fifth graders' understanding of fractions and division accurately predicts their high school competency in general math achievement.

The researchers controlled for parents' education and income as well as for the students' gender, IQ and knowledge of addition, subtraction and multiplication. The study is in the journal *Psychological Science*.

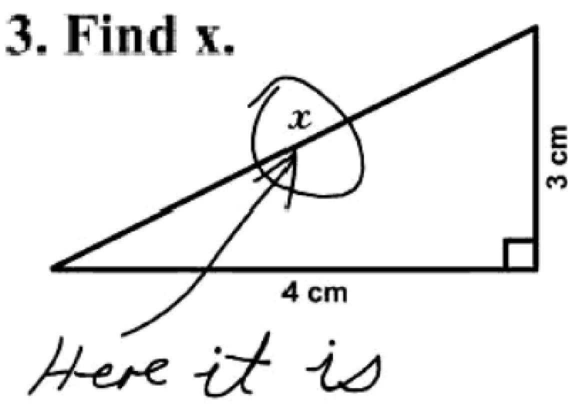
The researchers note that it's clear we need to improve instruction in fractions, ratios and proportions along with long division.

So let's get back to the fact that we know that 1/2 is equivalent to 3/6 is equivalent to 502/1004, which is equivalent to 9/18. And that is just a fraction of the hard work we need to do.

—Christie Nicholson

Lesson 3 Summary: January 31st
Tic Tac Toe
Prime Numbers
Plimpton 322
Pythagorean Theroem

Lesson Handouts
Tic-Tac-Toe Boards
Pythagorean Theorem worksheet (Kuta)
Areas and Triangles worksheet (Kuta)
Factorization worksheet (Kuta)



By the way, why do mathematicians wear glasses?

Let's go over the problem at the end of last class!

1/x + 1/y + 1/z =

The common denominator we can be sure about is simply x times y times z or xyz. Now, what must we multiply the first fraction to yield xyz. Well, it would be yz. We then mulltiply the numerator (1) by yz. We repeat this process for the remaining two fractions to get a final answer of:

(yz + xz + xy)/xyz

WARM UP

1. What is a unit fraction, give an example.

2. 1/2 + 2/3 + 3/4 =

3. 1/2 $\div$ 2/5 =

4. 1/a + 2/b + 3/c =

5. What is all the fuss with those Egyptian pyramids anyway?

Tic Tac Toe

This game is considered to be the oldest game ever devised by humans still played today. It is also the first computer game ever built! Archeologists can trace 3-in-a-row games to the ancient Egyptians of 1300 B.C. During the time of the great pharaohs, *games such as TTT were an important part of everyday life*.

Pair up, play 9 games, keep track of wins and losses, and try to think of strategies for winning. Imagine yourself in ancient Egypt, playing under a hot sun, a half finished pyramid visible in the distance.

Consider: If you consider the first move as having 9 possible ways to play, then the second move 8 possible ways and so on, there are 9! ways to play or ***362,880 possibilities****.*

X O

W	L	D
2	1	6
4	2	3
2	1	6
3	1	5
4	1	4

Tic-Tac-Toe Strategies:

There are basically only three main Tic-Tac-Toe strategies you must follow to win:

Choose to start: If you have the choice you should always start because you mathematically have more options to win.

If you are first, choose a corner. You should always choose a corner first because mathematically it gives the opponent the least amount of options to avoid you winning.

If you are second, always choose corner or center choose center if available, if not choose corner. You will lose if you do not follow this strategy!

If both players follow these rules and pay attention during the rest of the game, neither will win and neither will lose. (called a draw or cat's game) However, most don't pay attention and you will find opportunities to win.

Before we move on from the Rhind Papyrus, let's consider one other interesting concept that the ancient Egyptians appeared to have been aware of on some level.

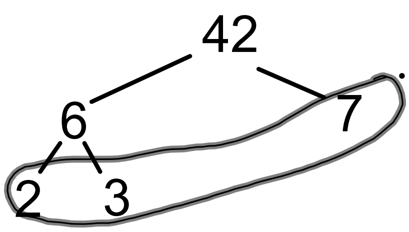
That is the notion of **Prime Numbers**. Prime numbers are a big deal, so much so that we will cover them a few more times in this course. However, today, let's just focus on their definition, relationship to composites and how to prime factorize.

Prime Numbers *are natural numbers greater than 1 that are only divisible by 1 and that number.*

For example, 5 is prime because only 1 and 5 divide it. As a counter example, 6 is not prime because while 1 and 6 divide it, so do 2 and 3. Numbers that are not prime are called **composite numbers**. The interesting thing about composite numbers is that they are made up of prime numbers.

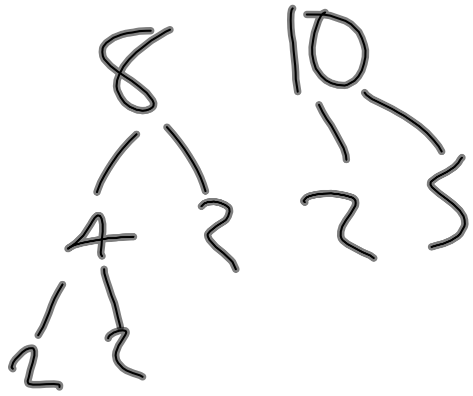
If we start breaking down a composite number, we will ALWAYS get a series of prime numbers whose product is that composite number. This process of finding the "ingredient" numbers of a composite is called prime factorization.

Let's try one together and then you can attempt the worksheet.



Once we have only prime numbers, we write our answer from smallest to greatest, so the prime factorization of 42 = 2*3*7.

1) 1, 2, 3, 5, 6, 10, 15, 30



So far, we have discussed the *Ishango Bone*, and we delved into the *Rhind Papyrus*, two important and ancient mathematical texts.

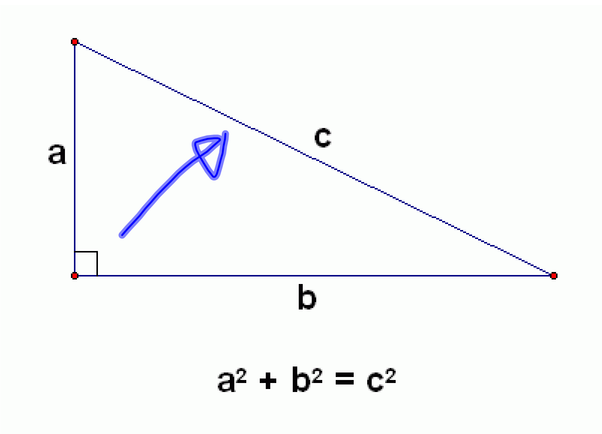
Another artifact we will consider today is the **Plimpton 322**. This is a mysterious clay tablet written around 1800 B.C. This tablet is traced to the Old Babylonian civilization, what is now modern day Iraq.

It is named Plimpton after a New Yorker, George Plimpton, who purchased it for 10 lousy bucks in 1922!
Lesson: Once in a while, something that appears worthless to everyone is actually a treasure in disguise.

What is of interest to us is the list of **Pythagorean Triples** found on the Plimpton 322.
Before we discuss what they are, and how they are important, we should first consider the formula that allows us to solve for them....
...perhaps the most famous math equation this little rock we fondly call planet earth has ever known, that is...

...of course the **Pythagorean Theorem**!

Formally, it states that *for any right triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the remaining two smaller sides.*
This is often expressed as:



To be fair, there is evidence that the theorem was developed by the Hindu Mathematician Baudhayana hundreds of years before Pythagoras.

<u>Lesson:</u> *In history, as in life, perception is reality. You might discover something amazing, but the world might just give someone else the credit.*

Hey! This is math class after all, so let's dive into some more problems.

However, since the Pythagorean Theorem is so straightforward, the following worksheet asks for the area of the triangles. Let's refresh what that formula is...

Area of any Triangle = (1/2) x Base x Height

Pythagorean Triples

A Pythagorean triple consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. Such a triple is commonly written (a, b, c), and a well-known example is (3, 4, 5).

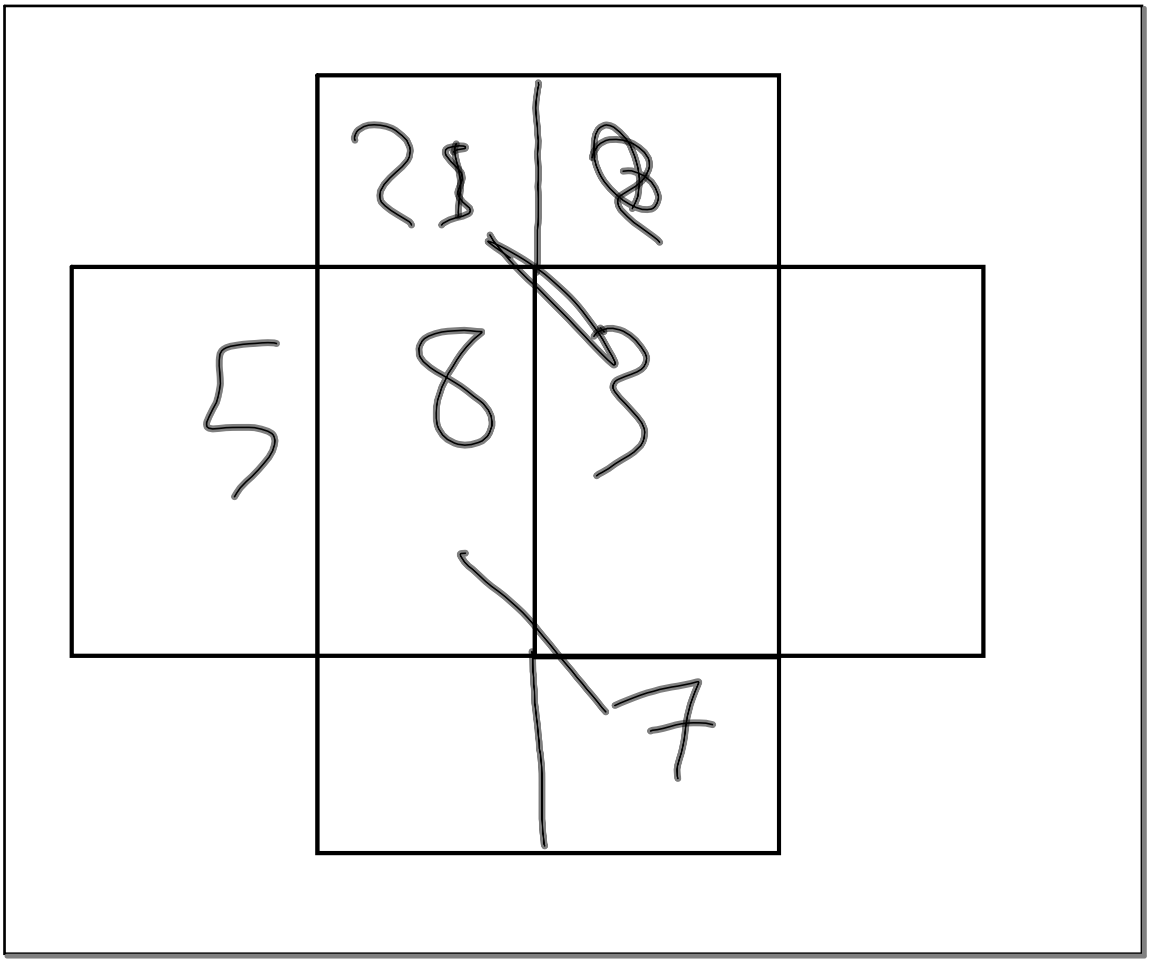
However, right triangles with non-integer sides do not form Pythagorean triples. For instance, the triangle with sides a = b = 1 and c = √2 is right, but (1, 1, √2) is not a Pythagorean triple because √2 is not an integer.

The triples listed on the Plimpton 322 are too many to have been constructed by brute force. From a modern perspective, a method for constructing such triples is a **significant early achievement**, known before only among the Greeks.

HW for Tuesday
LOGIC PUZZLE:

On the following page, there will be a cross shaped design with 8 boxes. The goal of the game is using the numbers 1-8 only, arrange the numbers in the box however you like, the only condition is no two adjacent numbers can touch, vertically, horizontally, or diagonally. So, for example, 4 and 5 should be no where near one another!

Try to think through the logic of the game before you start!


21
8
5
8
3
7

Tic-Tac-Toe

A game for two players.

Using a traditional three-by-three tic-tac-toe grid, players alternate writing an "X" or "O" on the board. Traditionally the player using X goes first. The goal for each player is to get three of their symbols in a row, vertically, horizontally, or diagonally. The first player to do so is the winner. Strategies include blocking your opponent to keep them from completing their three-in-a-row symbols.

Tic-Tac-Toe

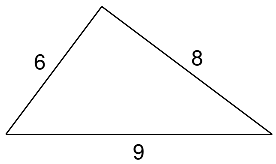
Name________________________________

The Pythagorean Theorem

Date________________ Period____

Do the following lengths form a right triangle?

1)



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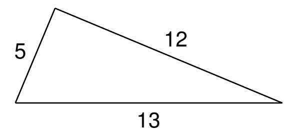

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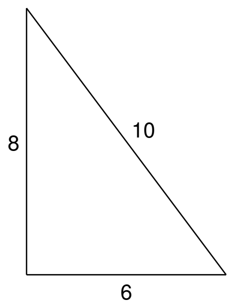

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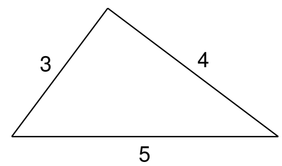
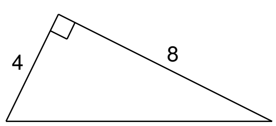


5) $a = 6.4,\ b = 12,\ c = 12.2$

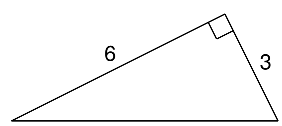
6) $a = 2.1,\ b = 7.2,\ c = 7.5$

Find each missing length to the nearest tenth.

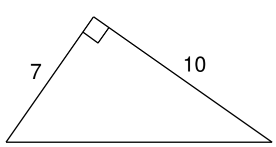
7)



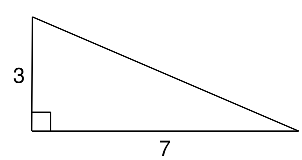
8)



9)



10)



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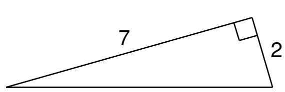

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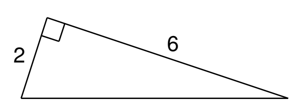

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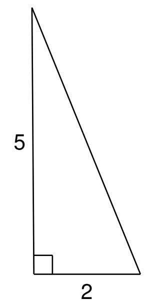

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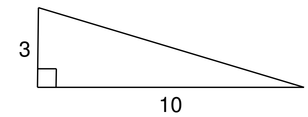

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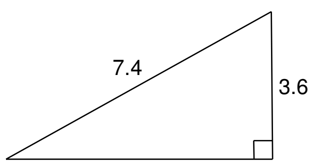

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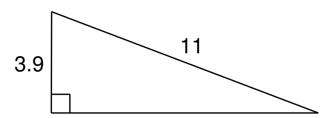

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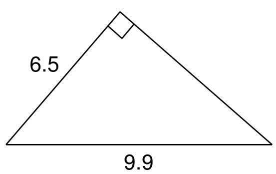

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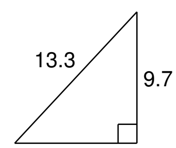

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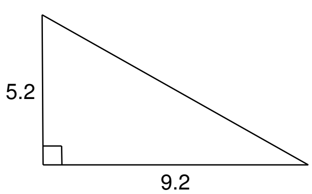

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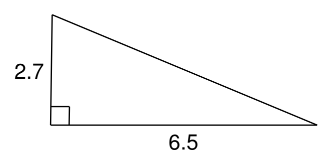

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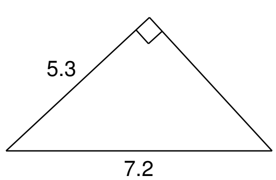

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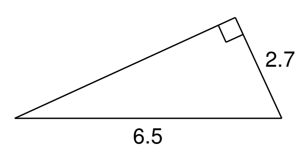

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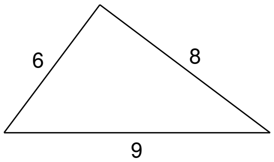
Name_______________________________

The Pythagorean Theorem

Date________________ Period____

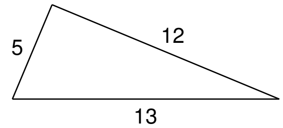
Do the following lengths form a right triangle?

1)



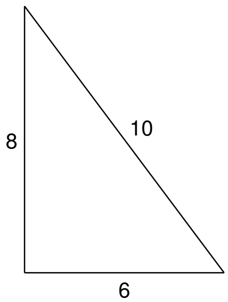
No

2)



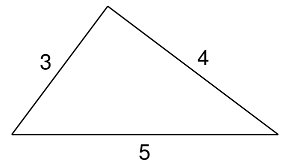
Yes

3)



Yes

4)



Yes

5) $a = 6.4, \; b = 12, \; c = 12.2$

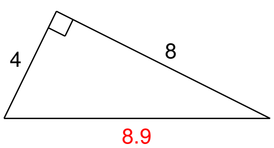
No

6) $a = 2.1, \; b = 7.2, \; c = 7.5$

Yes

Find each missing length to the nearest tenth.

7)



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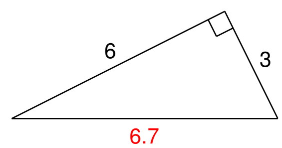

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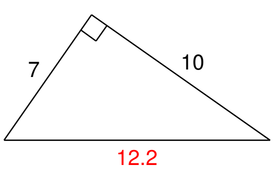

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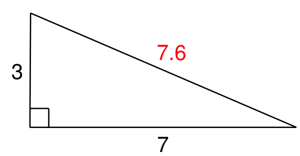

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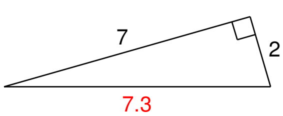

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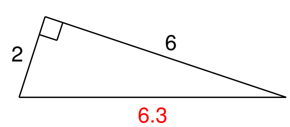

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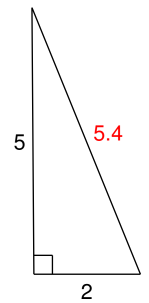

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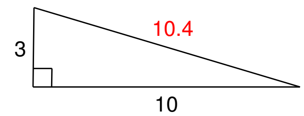

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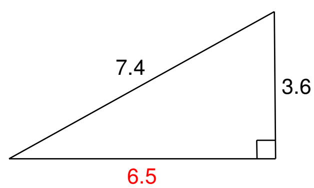

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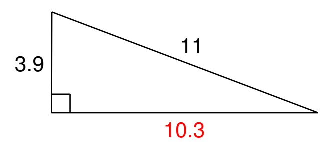

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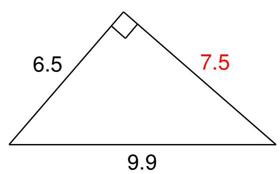

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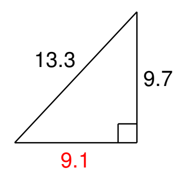

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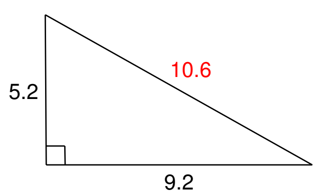

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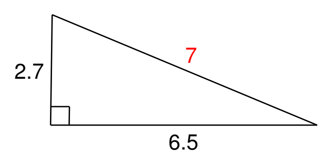

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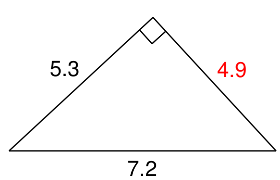

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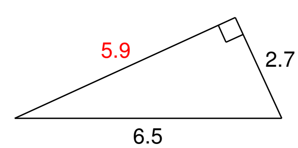

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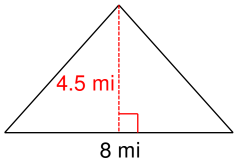
Name_______________________

Area of Triangles

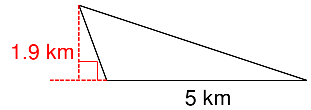
Date________________ Period____

Find the area of each.

1)



2)



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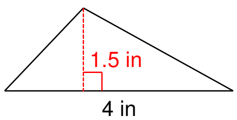

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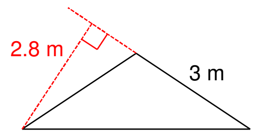

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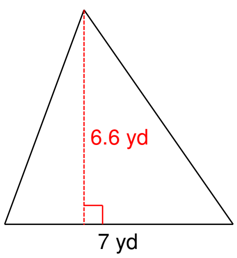

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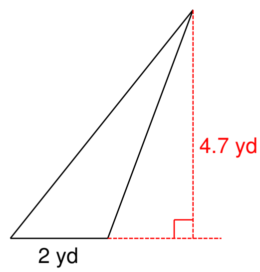

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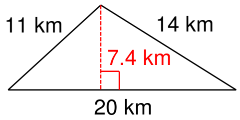

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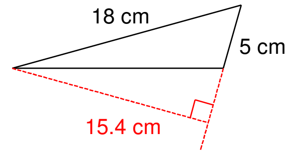

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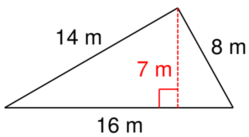

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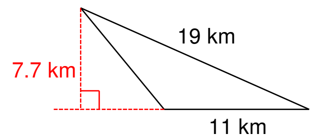

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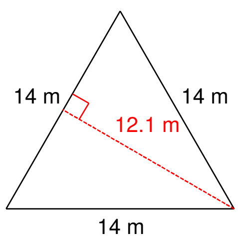

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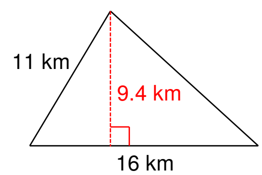

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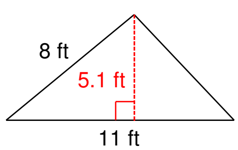

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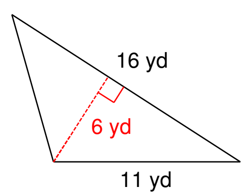

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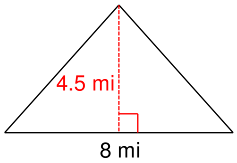
Name________________________

Area of Triangles

Date________________ Period____

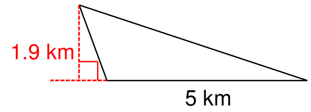
Find the area of each.

1)



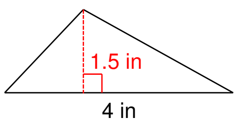
18 mi^2

2)



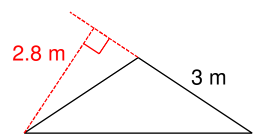
4.75 km^2

3)



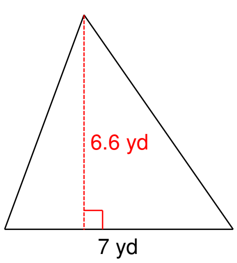
3 in^2

4)



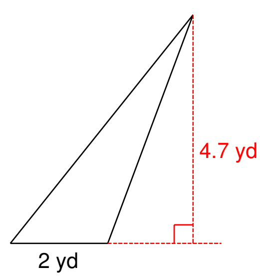
4.2 m^2

5)



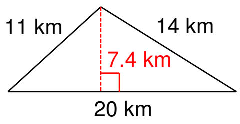
23.1 yd^2

6)



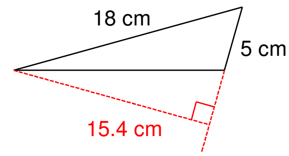
4.7 yd^2

7)



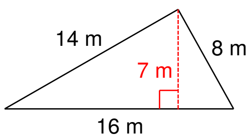
74 km²

8)



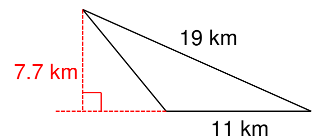
38.5 cm²

9)



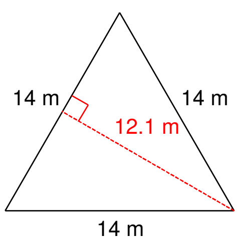
56 m²

10)



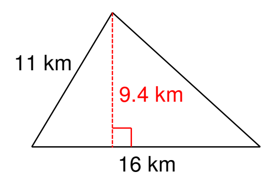
42.35 km²

11)



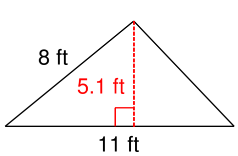
84.7 m²

12)



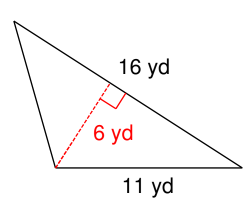
75.2 km²

13)



28.05 ft²

14)



48 yd²

 Name________________________________

Factors and Factorization

Date________________ Period____

List all positive factors of each.

1) 30

2) 22

3) 28

4) 16

5) 60

6) 87

7) 68

8) 99

9) 85

10) 72

11) 96

12) 74

13) 86

14) 75

Write the prime-power factorization of each.

15) 48

16) 35

17) 46

18) 40

19) 100

20) 66

21) 75

22) 72

23) 65

24) 81

25) 80

26) 54

27) 972

28) 660

Name________________________________

Factors and Factorization

Date________________ Period____

List all positive factors of each.

1) 30

1, 2, 3, 5, 6, 10, 15, 30

2) 22

1, 2, 11, 22

3) 28

1, 2, 4, 7, 14, 28

4) 16

1, 2, 4, 8, 16

5) 60

1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

6) 87

1, 3, 29, 87

7) 68

1, 2, 4, 17, 34, 68

8) 99

1, 3, 9, 11, 33, 99

9) 85

1, 5, 17, 85

10) 72

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

11) 96

1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

12) 74

1, 2, 37, 74

13) 86

1, 2, 43, 86

14) 75

1, 3, 5, 15, 25, 75

Write the prime-power factorization of each.

15) 48

$2^4 \cdot 3$

16) 35

$5 \cdot 7$

17) 46

$2 \cdot 23$

18) 40

$2^3 \cdot 5$

19) 100

$2^2 \cdot 5^2$

20) 66

$2 \cdot 3 \cdot 11$

21) 75

$3 \cdot 5^2$

22) 72

$2^3 \cdot 3^2$

23) 65

$5 \cdot 13$

24) 81

3^4

25) 80

$2^4 \cdot 5$

26) 54

$2 \cdot 3^3$

27) 972

$2^2 \cdot 3^5$

28) 660

$2^2 \cdot 3 \cdot 5 \cdot 11$

<u>Lesson 4 Summary: February 5th</u>
Probability Introduction (several revisits during course)
A proof of Pythagorean Theorem
Magic Squares

<u>Lesson Handouts</u>
Dice (Cliff Pickover, The Math Book)

“The highest form of pure thought is in mathematics.”

Plato

What did the zero say to the 8?

WARM UP

$A = \frac{1}{2}bh$

1. What is the base (length) of a triangle with an area of 96 and height 16?

$96 = \frac{1}{2} \cdot b \cdot 16$

2. Is 109 prime or composite? How about 233?

3. Prime factorize the number 256 and then the number 555.

$a^2 + 21^2 = 35^2$

4. If you know the hypotenuse of a right triangle is of length 35, and you know the other side is of length 21, what is the length of the third side?

$a^2 + 441 = 1225$

$a^2 = 784$

$\sqrt{a^2} = \sqrt{784}$

$a = 28$

Pythagorean Triples

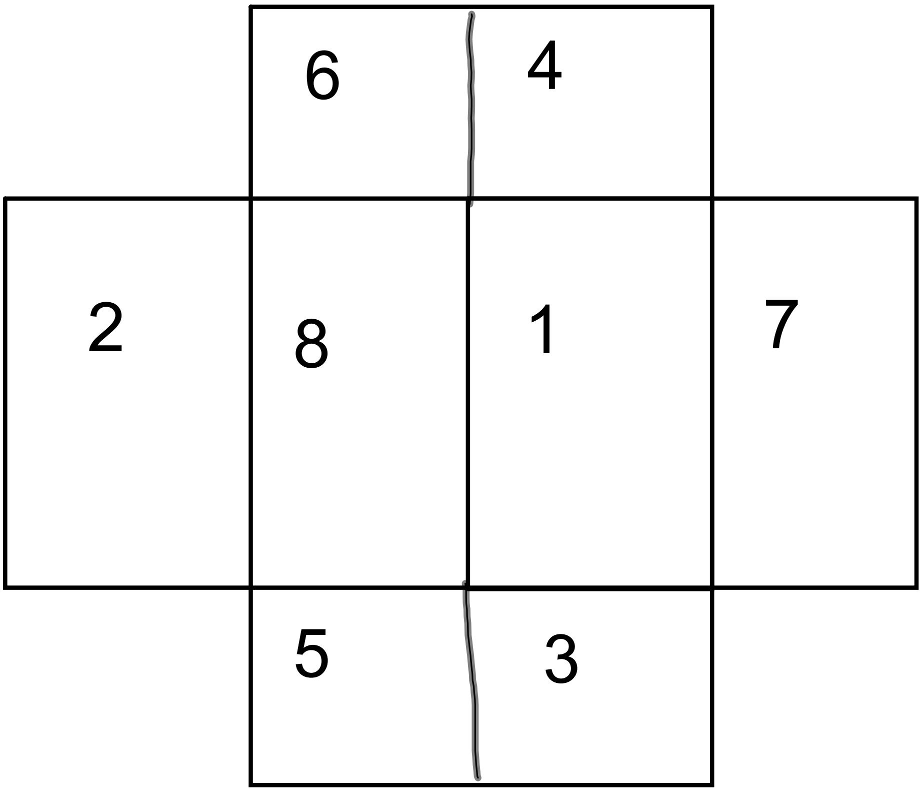
A Pythagorean triple consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. Such a triple is commonly written (a, b, c), and a well-known example is (3, 4, 5).

However, right triangles with non-integer sides do not form Pythagorean triples. For instance, the triangle with sides a = b = 1 and c = √2 is right, but (1, 1, √2) is not a Pythagorean triple because √2 is not an integer.

The triples listed on the Plimpton 322 are too many to have been constructed by brute force. From a modern perspective, a method for constructing such triples is a **significant early achievement**, known before only among the Greeks.

SOLUTION for LOGIC PUZZLE:

There are many correct solutions. However, since 1 and 8 are only adjacent to 2 and 7, respectively, we want those to be placed where they touch the most squares.



What is randomness?

How do we define it?

Can we create it?

One way in which we can create some reliable randomness is the rolling of dice. Since the dice is presumed to be of evenly distributed weight, there is an equal likelihood of the die landing on any of its 6 given sides. This allows us to use dice as random generators and why it can appear in games where randomness is needed.

For random generators like dice, we often want to calculate probabilities, and the key is to consider the overall sample space and then consider the desired result.
The answer is:

desired result/sample space

Consider: What is the probability of getting a ace or black card on a single draw of a regular deck of cards?

ANS: 52 is total sample space, desired result is 26 black cards + 2 red aces =

28/52

Now try for fun:

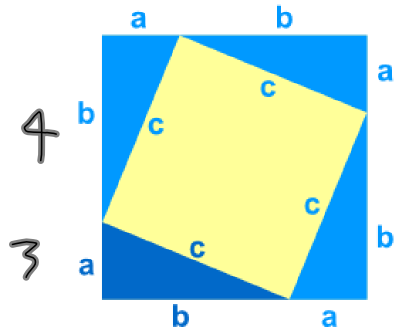
What is the probability of getting a two digit sum on a single roll of a pair of dice?

There are 6x6 = 36 possible outcomes.
The highest sum is 6 and 6 or 12.
The second highest is 11, which can be obtained by a 5 and 6 or a 6 and 5.
The third highest is a 10, which can be obtained by a 5 and 5, a 4 and 6 or a 6 and 4.
Together, we have 6 total outcomes that have a two digit sum so our answer is 6/36 or 1/6.

	1	2	3	4	5	6
1	2	3				
2	3					
3						
4						
5						
6						

One of the reasons why mathematics is so powerful is that anything (even 1+1=2) has to be **proven** in order for it to become accepted as fact. This process of writing a proof is at the heart of mathematics, and can be surprisingly difficult to do, even for simple proofs. In addition, there can be more than one proof for the same concept, for example there is a book that outlines 367 distinct proofs for the Pythagorean Theorem. Let's briefly take a look at one of these proofs.

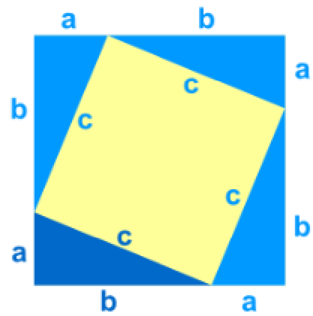
This proof will use basic algebra to ***prove*** that $a^2 + b^2 = c^2$.
Consider the following diagram:



What is the area of the bigger square?

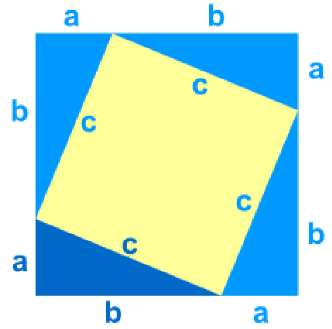
The area is given by:
A = (a+b)(a+b)

What about the area of the smaller yellow square?



Well, that is just $A = c^2$.
How about the area of the four blue triangles?
Well if each triangle is given by 1/2ab, then all 4 is 4*1/2ab or simplifies to 2ab.

Therefore, the sum of the yellow square and the four triangles is given by $c^2 + 2ab$.



Now we have two distinct ways to represent the overall area:
$(a+b)(a+b)$ and $c^2 + 2ab$

Setting them equal to one another
$(a+b)(a+b) = c^2 + 2ab$

From here, use FOIL, and algebraic rearranging to see if you can arrive at the Pythagorean Theorem. If you can, then we have proven the Theorem. **Q.E.D.**
This is a proof from China from over 2000 years ago!

$$(a+b)(a+b) = c^2 + 2ab$$

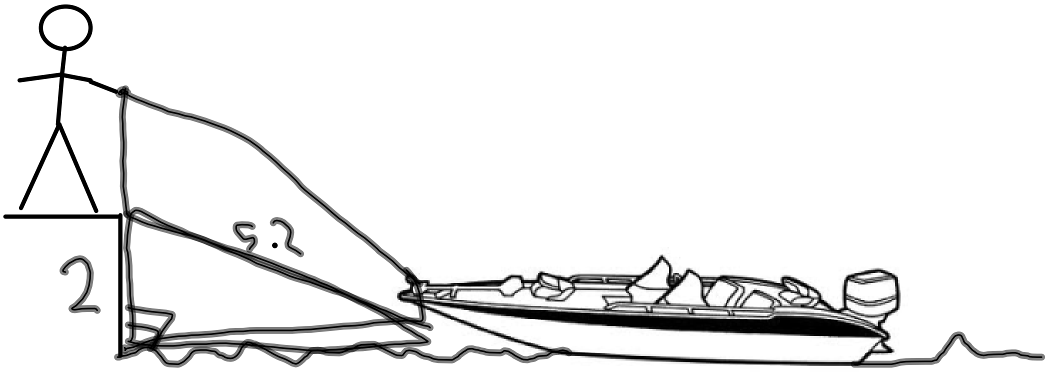
$$a^2 + ab + ba + b^2 = c^2 + 2ab$$

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

$$-2ab \qquad\qquad -2ab$$

PYTHAGOREAN THEOREM CHALLENGE:

Gary is standing on a dock 2m above the water. He is pulling in a boat that is attached to the end of a 5.2m rope, if he pulls in 2.3m of rope, how far did he move the boat?



ANS:

First, we calculate the starting distance the boat is from the dock. This is given by $2^2 + b^2 = 5.2^2$. Solving for b, we get 4.8m.
Now, since we know we moved 2.3m of the rope in, we take the original rope length of 5.2m and subtract 2.3m to get 2.9m of rope after Gary pulls the boat in. Solving again, we have $2^2 + b^2 = 2.9^2$. Solving for b, we get b = 2.1m.

The answer is then just the original distance of 4.8 minus the new distance of 2.1 to get **2.7m**.

Magic squares

Magic squares are boxes, where the sums of the numbers in the horizontal rows, vertical columns and main diagonals are all equal. Magic squares have been mentioned in ancient Chinese literature since 2200 B.C. Here we find the following story:

1-9 (SUM) → 15

At one time, there was a huge flood. The people tried to offer sacrifices to the god of one of the flooding rivers, the Lo river, to calm his anger. As they were doing this, a turtle emerged from the water with a curious pattern on its shell, with patterns of circular dots arranged in a three-by-three grid on the shell, such that the sum of the numbers in each row, column and diagonal was the same: 15. The people were able to use this magic square to control the river and reduce the flood.

8	3	4
		6
		5

People normally say there is only one 3x3 magic square. In one sense this is true, in another it is not. It is true because all the 3x3 magic squares are related by symmetry. Once you have one, you can get all the others by turning or flipping the one you found.

1	2	3
4	5	6
7	8	9

3 2 1
6 5 4
4 8 7

HW:

Complete the 3x3 magic square and then list all the other symmetries of the square.

HINT: You should have a total of 8 magic squares.

Dice

c. 3000 B.C.

Imagine a world without random numbers. In the 1940s, the generation of statistically random numbers was important to physicists simulating thermonuclear explosions, and today, many computer networks employ random numbers to help route Internet traffic to avoid congestion. Political poll-takers use random numbers to select unbiased samples of potential voters.

Dice, originally made from the anklebones of hoofed animals, were one of the earliest means for producing random numbers. In ancient civilizations, the gods were believed to control the outcome of dice tosses; thus, dice were relied upon to make crucial decisions, ranging from the selection of rulers to the division of property in an inheritance. Even today, the metaphor of God controlling dice is common, as evidenced by astrophysicist Stephen Hawking's quote, "Not only does God play dice, but He sometimes confuses us by throwing them where they can't be seen."

The oldest-known dice were excavated together with a 5,000-year-old backgammon set from the legendary Burnt City in southeastern Iran. The city represents four stages of civilization that were destroyed by fires before being abandoned in 2100 B.C. At this same site, archeologists also discovered the earliest-known artificial eye, which once stared out hypnotically from the face of an ancient female priestess or soothsayer.

For centuries, dice rolls have been used to teach probability. For a single roll of an n-sided die with a different number on each face, the probability of rolling any value is $1/n$. The probability of rolling a particular sequence of i numbers is $1/n^i$. For example, the chance of rolling a 1 followed by a 4 on a traditional die is $1/6^2 = 1/36$. Using two traditional dice, the probability of throwing any given sum is the number of ways to throw that sum divided by the total number of combinations, which is why a sum of 7 is much more likely than a sum of 2.

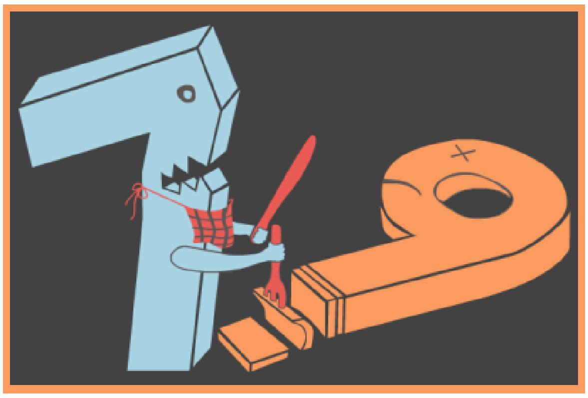
SEE ALSO Law of Large Numbers (1713), Buffon's Needle (1777), Least Squares (1795), Laplace's *Théorie Analytique des Probabilités* (1812), Chi-Square (1900), Lost in Hyperspace (1921), The Rise of Randomizing Machines (1938), Pig Game Strategy (1945), and Von Neumann's Middle-Square Randomizer (1946).

Dice were originally made from the anklebones of animals and were among the earliest means for producing random numbers. In ancient civilizations, people used dice to predict the future, believing that the gods influenced dice outcomes.

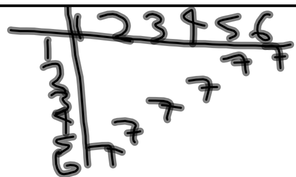
Lesson 5 Summary: February 7th
An example of a false proof
Pythagoras
Greeks, Zeno
Euclid

Lesson Handouts
New Prime Number found article (CNET)
Solving Proportions worksheet (Kuta)
Multi step Equations worksheet (Kuta)

Why was 6 afraid of 7?



WARM UP:



1. Give a few examples of how to generate randomness or what you define as random. $\frac{6}{36}$

2. What is the probability of getting a sum of 7 of a single roll of a pair of dice?

3. Is 5,12,14 a Pythagorean triple? How about 7,24,25?

4. What is the chance of getting 4 heads in a row?

PROOF THAT 2 = 1

1. Let a and b be equal non-zero quantities and **$a = b$**

2. Multiply through by a: **$a^2 = ab$**

3. Subtract b^2: **$a^2 - b^2 = ab - b^2$**

4. Factor both sides: **$(a - b)(a + b) = b(a - b)$**

5. Divide out (a - b): **$a + b = b$**

6. Observing that a = b: **$b + b = b$**

7. Combine like terms on the left: **$2b = b$**

8. Divide by the non-zero b: **$2 = 1$**

Q.E.D.

Spend a few minutes and try to isolate the single mistake in the proof!

MAGIC SQUARES SOLUTION

Here's how to do it:

x 1 x ----- x 1 x ----- x 1 x
x x x ----- x x x ----- 3 x x
x x x ----- x x 2 ----- x x 2

x 1 x ----- x 1 x ----- x 1 6
3 x x ----- 3 5 x ----- 3 5 x
4 x 2 ----- 4 x 2 ----- 4 x 2

x 1 6 ----- 8 1 6 ----- 8 1 6
3 5 7 ----- 3 5 7 ----- 3 5 7
4 x 2 ----- 4 x 2 ----- 4 9 2

8	1	6
3	5	7
4	9	2

1. Start from top row middle column with S=1, S is the starting value.
2. Add 2,3,4,5,6,7,8,9 diagonally, that means try to go UP then RIGHT.
3. If UP is blocked, go to Right column, and fill the bottom row. eg. like filling "2".
4. If RIGHT is blocked, go to the LEFT most column of one row above, eg like filling "3".
5. Since this is a 3x3 square, after 3 numbers, then fill the next number just one row DOWN.
eg like filling in "4".
6. Repeat from step 3.

Pythagoras and Mathematical Religiosity

We have now spent some time with the Pythagorean Theorem. Let us also consider Pythagoras himself. As a Greek mathematician living about 500 B.C., numbers were gods, pure and free from the constraints of the real world. Pythagoras and his peers felt that numbers were alive. They found mathematics to be a revelation.

A question to ponder: is mathematics a creation of the human mind or is it a part of the universe that is independent of human thought?

Forget Waldo, where is Pythagoras?

THE ANCIENT GREEKS:

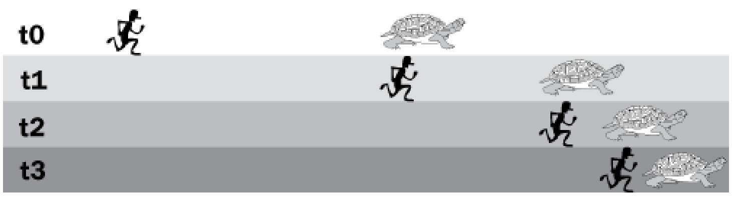
They revered mathematics more than modern modern Americans revere Justin Bieber & Lady Gaga, combined. Imagine that!

The Greeks: So much math, so little time....

Let's start our foray of ancient Greek mathematics by looking at a paradox. Specifically, it is one of several called Zeno's Paradoxes, named after a Greek Philosopher of the same name who posed a series of riddles.

So, imagine you are trying to get from Point A to Point B. For example, wherever you are to the closest wall or Philadelphia to some city you want to visit.

Well, one way to get from one point to the next you must first travel half of the distance. From there you must travel half of the remaining distance. This process continues....



<u>The Math behind Zeno's Paradox:</u>

1. To go half the distance from point A to point B is 1/2.
2. Now, at the halfway point between A and B, you again go half the distance. This fraction is now 1/4 since we are going half of what is left, which is half of 1/2.
3. From this second new point, we again go half, this time 1/8.
4. Now you might see the pattern..1/2 + 1/4 + 1/8 + 1/16 and so on..

What is interesting is that while the fractions get smaller and smaller, you never stop finding a distance that you need to half, so in that sense while you traverse smaller and smaller distances, **mathematically you will never arrive at your destination! The question (paradox) is how you can <u>ever</u> possibly finish traversing an infinite number of steps, no matter how small the steps may be.**

<u>A standout among s</u>
<u>Euclid and the Elem</u>

Euclid was one of the influential mathemati in the history of our p and he lived from arc 325 B.C. to 270 B.C.

He wrote one of the r influential math tome of all time, called the Elements.

The Elements:

- 13 books, published circa 300 B.C.
- It is a collection of definitions, postulates (axioms), propositions (theorems and constructions), and mathematical proofs of the propositions.
- It has influenced the development of logic and modern science.
- Some argue it is the second most influential book ever, only bested by the Bible.
- The Elements is very likely to be the contributions of many mathematicians and scholars of the time.
- Highlights of the contents include the Pythagorean Theorem, properties of triangles and circles, finding square roots, prime numbers, ratios and proportions, geometric series.

Now time to do some math!

Let's spend some time and make sure our fundamental skills are solid. It is only from being successful with the basics that we can hope to understand more complicated ideas.

As you work on these problems, imagine yourself living in a time where there are no computers, phones, tvs, cars or planes. The world is a simpler place, and the math you do can have profound implications for future generations.

10) $73 = {}^{-}6k + 42 + 6k + 30$

$73 \neq 72$ NO sol.

11) $-9 + 4r = {}^{-}4r - 9$

One of Euclid's many mathematical achievements was his proof that the number of primes is infinite. This fact still holds up today, in fact it even made the news this week, sort of.... First, the proof.

Proof

Let us suppose that p 1 , p 2 , p 3 , ... p n are prime numbers. Multiply them together and add 1, calling this number a new integer q . If q is a prime number, then we have a new prime. If q is not a prime, it must be divisible by a prime number r . But r cannot be p 1 or any other from our original list of prime numbers, because if you divide q by any of p 1 , p 2 , p 3 , ... p n you will get a remainder 1, which means that q is not divisible by any of these prime numbers. So r is a new prime. Whichever way you choose to look at it, either you have found a new prime q, or if q is not a prime, than you have found that it has a new prime for a prime factor.

Math in the News

See the handout on the latest and largest prime number discovered. Remind yourself of what a prime number is, then ask how big is it? Try over **17 million digits long**!

For some perspective, the absolutely huge number one billion is of course written as 1,000,000,000, which is a paltry 10 digits long!

Homework Assignment:

Due Tuesday, February 12th, 2013.

1. Find any mathematician or mathematical development, the limiting criteria is it must be from Euclid's time or before only!

2. Research the specific contributions and write at least a one page paper (typed only) that details why this mathematician/mathematical discovery was influential.

3. Make sure you include at least one example of the actual mathematics in your paper and make sure it is an example you yourself understand!

4. You will be graded on: original voice (beware copy and pasters!), grammar and spelling, adhering to the above rules.

<u>Possible Topics/Mathematicians to consider:</u>

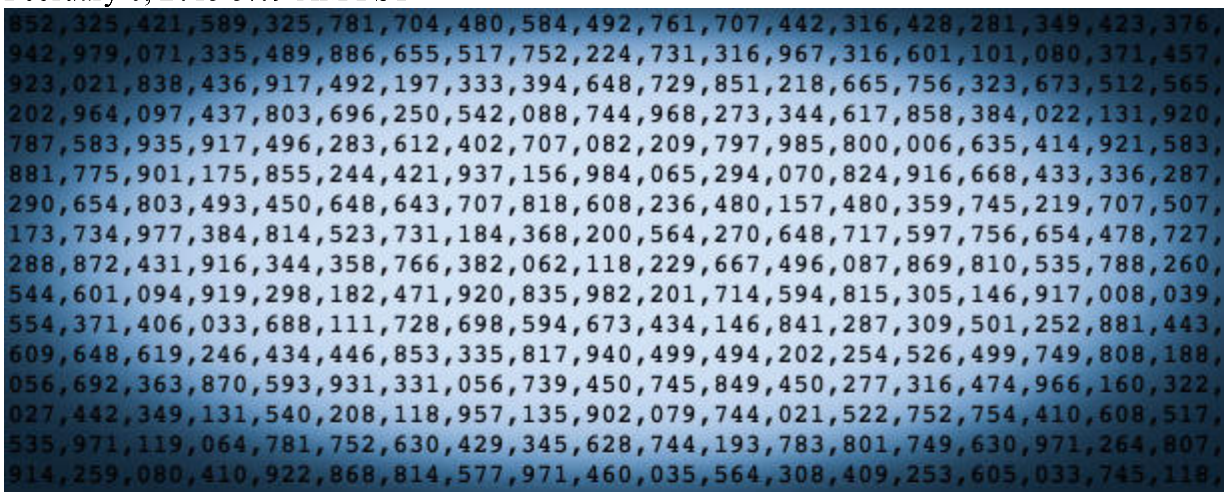
Euclid
Zeno
Thales of Miletus
Pythagoras of Samos
Aristotle
Archimedes

Early Egyptian Mathematics
Mathematical Artifacts (Rhind Papyrus, Ishango Bone, etc)
Antique Number Systems (Mayan, Babylonian, etc)

Amateur effort finds new largest prime number

A Missouri professor, one of a team of nearly 100,000 volunteers, has found a highly unusual 17-million digit number -- and brought a prime-hunting project closer to a $150,000 prize.

by Stephen Shankland
February 6, 2013 3:09 AM PST



A tiny portion of the 48th Mersenne prime, a number more than 17 million digits long. Written as text, the entire number is a 22.5MB file.

The Great Internet Mersenne Prime Search (GIMPS) project has scored its 14th consecutive victory, discovering the largest prime number so far.

The number, 2 to the power of 57,885,161 minus 1, is a digit that's 17,425,170 digits long. That's big enough that if you want to see the full text, you'll have to brace yourself for a 22.5MB download.

GIMPS, a cooperative project splitting the search across thousands of independent computers, announced the find yesterday after it had been confirmed by other checks. At present, there are 98,980 people and 574 teams involved in the GIMPS project; their 730,562 processors perform about 129 trillion calculations per second.

The project has a lock on the market for mongo new prime numbers. The discoverer of this particular prime is Curtis Cooper, a professor at the University of Central Missouri who runs the prime-hunting software on a network of computers and who's found record primes in 2005 and 2006. It's not just his effort that's important, though; it relied also on others' machines ruling out other candidates.

A prime number is divisible only by itself and the number 1. Once a mathematical curiosity, primes now are crucial to encrypted communications. Mersenne primes are named after Marin Mersenne, a French monk born in 1588 who investigated a particular type of prime number: 2 to the power of "p" minus one, in which "p" is an ordinary prime number.

Cooper's find is the 48th Mersenne prime so far discovered. GIMPS has found the 14 largest Mersenne primes, the organization said.

Discovering Mersenne primes is not a get-rich-quick scheme, though Cooper won a $3,000 prize. It could be more lucrative at some point: An Electronic Frontier Foundation award of $150,000 will go to the discoverer of the first prime number with at least 100 million digits. It's already awarded prizes for primes 1-million and 10-million digits, and it's got a $250,000 prize queued up for a billion-digit prime.

GIMPS is steadily advancing on the bigger numbers.

In 1998, the project found 2^3021377-1, a number 909,526 digits long. By 2001, GIMPS found the 39th Mersenne prime, a number 4,053,946 digits long. The 43rd Mersenne prime, which Cooper's effort found, is a 9,152,052-digit numeral.

Searching for prime numbers is a project that can easily be split across countless computers through an idea called distributed computing. Not all computing chores are so amenable to cooperation, though.

Some of those labors, such fluid dynamics research that can be used to model nuclear weapons explosions or car aerodynamics, can be run on closely independent computing nodes connected by a high-speed network.

Other computing chores can't be broken down into parallel tasks at all, a problem given that power-consumption limits stalled processor clock speed increases in recent years.

A computer-science idea called Amdahl's Law, named after mainframe computer designer Gene Amdahl, shows the limits of parallel computation. If some portion of a computer program can't be sped up by parallel processing, at a certain point throwing more processors at the problem will stop producing any speedup in the computation.

Name____________________________________

Solving Proportions

Date__________________ Period____

Solve each proportion.

1) $\frac{10}{8} = \frac{n}{10}$

2) $\frac{7}{5} = \frac{x}{3}$

3) $\frac{9}{6} = \frac{x}{10}$

4) $\frac{7}{n} = \frac{8}{7}$

5) $\frac{4}{3} = \frac{8}{x}$

6) $\frac{7}{b + 5} = \frac{10}{5}$

7) $\frac{6}{b - 1} = \frac{9}{7}$

8) $\frac{4}{m - 8} = \frac{8}{2}$

9) $\frac{5}{6} = \frac{7n + 9}{9}$

10) $\frac{4}{9} = \frac{r - 3}{6}$

11) $\frac{7}{9} = \frac{b}{b - 10}$

12) $\frac{9}{k - 7} = \frac{6}{k}$

13) $\frac{4}{n + 2} = \frac{7}{n}$

14) $\frac{n}{n - 3} = \frac{2}{3}$

15) $\frac{x - 3}{x} = \frac{9}{10}$

16) $\frac{5}{r - 9} = \frac{8}{r + 5}$

17) $\frac{p + 10}{p - 7} = \frac{8}{9}$

18) $\frac{2}{8} = \frac{n + 4}{n - 4}$

19) $\frac{n - 5}{n + 8} = \frac{2}{7}$

20) $\frac{n - 6}{n - 7} = \frac{9}{2}$

Kuta Software - Infinite Algebra 1

Name________________________

Solving Proportions

Date________________ Period____

Solve each proposition.

1) $\frac{10}{8} = \frac{n}{10}$

{12.5}

2) $\frac{7}{5} = \frac{x}{3}$

{4.2}

3) $\frac{9}{6} = \frac{x}{10}$

{15}

4) $\frac{7}{n} = \frac{8}{7}$

{6.12}

5) $\frac{4}{3} = \frac{8}{x}$

{6}

6) $\frac{7}{b+5} = \frac{10}{5}$

{−1.5}

7) $\frac{6}{b-1} = \frac{9}{7}$

{5.66}

8) $\frac{4}{m-8} = \frac{8}{2}$

{9}

9) $\frac{5}{6} = \frac{7n+9}{9}$

{−0.21}

10) $\frac{4}{9} = \frac{r-3}{6}$

{5.66}

11) $\frac{7}{9} = \frac{b}{b-10}$

$\{-35\}$

12) $\frac{9}{k-7} = \frac{6}{k}$

$\{-14\}$

13) $\frac{4}{n+2} = \frac{7}{n}$

$\{-4.66\}$

14) $\frac{n}{n-3} = \frac{2}{3}$

$\{-6\}$

15) $\frac{x-3}{x} = \frac{9}{10}$

$\{30\}$

16) $\frac{5}{r-9} = \frac{8}{r+5}$

$\{32.33\}$

17) $\frac{p+10}{p-7} = \frac{8}{9}$

$\{-146\}$

18) $\frac{2}{8} = \frac{n+4}{n-4}$

$\{-6.66\}$

19) $\frac{n-5}{n+8} = \frac{2}{7}$

$\{10.19\}$

20) $\frac{n-6}{n-7} = \frac{9}{2}$

$\{7.28\}$

Kuta Software - Infinite Algebra 1 Name____________________

Multi-Step Equations

Date________________ Period____

Solve each equation.

1) $-20 = -4x - 6x$

2) $6 = 1 - 2n + 5$

3) $8x - 2 = -9 + 7x$

4) $a + 5 = -5a + 5$

5) $4m - 4 = 4m$

6) $p - 1 = 5p + 3p - 8$

7) $5p - 14 = 8p + 4$

8) $p - 4 = -9 + p$

9) $-8 = -(x + 4)$

10) $12 = -4(-6x - 3)$

11) $14 = -(p - 8)$

12) $-(7 - 4x) = 9$

13) $-18 - 6k = 6(1 + 3k)$

14) $5n + 34 = -2(1 - 7n)$

15) $2(4x - 3) - 8 = 4 + 2x$

16) $3n - 5 = -8(6 + 5n)$

17) $-(1 + 7x) - 6(-7 - x) = 36$

18) $-3(4x + 3) + 4(6x + 1) = 43$

19) $24a - 22 = -4(1 - 6a)$

20) $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$

Kuta Software - Infinite Algebra 1 Name________________________

Multi-Step Equations

Date________________ Period____

Solve each equation.

1) $-20 = -4x - 6x$

$\{2\}$

2) $6 = 1 - 2n + 5$

$\{0\}$

3) $8x - 2 = -9 + 7x$

$\{-7\}$

4) $a + 5 = -5a + 5$

$\{0\}$

5) $4m - 4 = 4m$

No solution.

6) $p - 1 = 5p + 3p - 8$

$\{1\}$

7) $5p - 14 = 8p + 4$

$\{-6\}$

8) $p - 4 = -9 + p$

No solution.

9) $-8 = -(x + 4)$

$\{4\}$

10) $12 = -4(-6x - 3)$

$\{0\}$

11) $14 = -(p - 8)$

$\{-6\}$

12) $-(7 - 4x) = 9$

$\{4\}$

13) $-18 - 6k = 6(1 + 3k)$

$\{-1\}$

14) $5n + 34 = -2(1 - 7n)$

$\{4\}$

15) $2(4x - 3) - 8 = 4 + 2x$

$\{3\}$

16) $3n - 5 = -8(6 + 5n)$

$\{-1\}$

17) $-(1 + 7x) - 6(-7 - x) = 36$

$\{5\}$

18) $-3(4x + 3) + 4(6x + 1) = 43$

$\{4\}$

19) $24a - 22 = -4(1 - 6a)$

No solution.

20) $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$

$\{-5\}$

Lesson 6 Summary: February 12th

Irrational Numbers & Hippasus
Perfect Squares
Sieve of Erastosthenes

Lesson Handouts

The Dangerous Ratio article (maths.org)
Sieve of Eratosthenes worksheet
Find the square root worksheet
Simplifying Square Roots worksheet (Kuta)

NEWS ALERT!

Teacher Arrested at JFK

A public school teacher was arrested today at John F. Kennedy International Airport this morning as he attempted to board a flight while in possession of a ruler, a protractor, a compass, a slide-rule and a calculator. At a press conference just before noon today, Attorney General Eric Holder said he believes the man is a member of the notorious Al-Gebra movement. Although he did not identify the man, he confirmed the man has been charged by the FBI with carrying weapons of math instruction.

'Al-Gebra is a problem for us', the Attorney General said. 'They derive solutions by means and extremes, and sometimes go off on tangents in search of absolute values.' They use secret code names like "X" and "Y" and refer to themselves as "unknowns" but we have determined that they belong to a common denominator of the axis of medieval with coordinates in every country. As the Greek philosopher Isosceles used to say, "There are 3 sides to every triangle." The Attorney General went on to say "Teaching our children sentient thought processes and equipping them to solve problems is dangerous and puts our government at risk."

Feb 6-9:18 AM

WARM UP:

1. What is the probability of getting a sum of 5 of a single roll of a pair of dice?

2. How do you define infinity?

3. Is math invented or discovered?

4. 5(2x + 1) - 3x = 8(x + 5) Solve for x.

5. What are the Elements? Why are they important?

6. EC: *From last week:* We know the probability of getting a sum of 7 on a single roll of the dice is 1/6. It has been said that to arrive at that result, you can consider any number on the first roll, therefore the answer is simply 1*(1/6) = 1/6. Does this strategy of making the first probability 1 work for the problem above, where the sum is 5? Why or why not?

Feb 6-9:21 AM

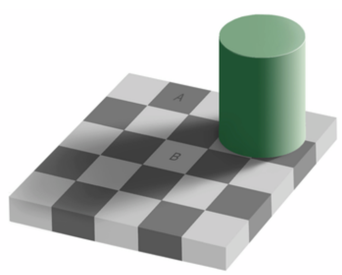
SUM Table for a single roll of a pair of Dice

	⚀	⚁	⚂	⚃	⚄	⚅
⚀				5		7
⚁			5		7	
⚂		5		7		
⚃	5		7			
⚄		7				
⚅	7					

Feb 12-10:18 AM

Optical Illusion Fun:

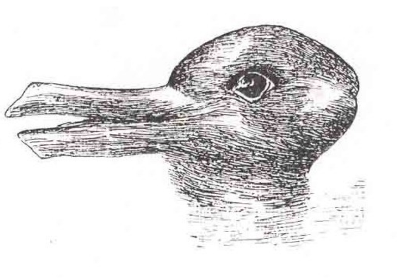
Which square is darker, A or B? Are they the same color?



http://en.wikipedia.org/wiki/Same_color_illusion

Feb 11-12:17 PM

Some see a rabbit, some see a duck.



Feb 12-10:13 AM

When knowledge is dangerous

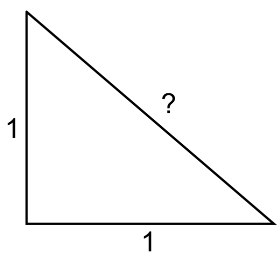
In today's information society, we are very accustomed to having an incredible amount of knowledge at our fingertips, in fact, we probably take it for granted.
One modern example of where information is still considered dangerous are resources like Wikileaks, where government and corporate secrets are revealed.

In the ancient Greek civilization, a possible parallel to Wikileaks is the story of Hippasus.

Hippasus was a philosopher and mathematician who is often credited with **the discovery that the square root of 2 is irrational**. The legend goes that this fact cost him his life!

Feb 12-10:41 AM

Please read this article on the story of Hippasus.
When you start the second page, please take a second and calculate the missing side using the Pythagorean Theorem.



Feb 12-12:26 PM

So what is an irrational number? This is a number that cannot be expressed as the ratio of two integers.

So for example, is 367.456 irrational?

No, since 367.456 can be rewritten as 367456/1000.

However, numbers like Pi are irrational. Another great example of irrational numbers are the square roots of numbers that are not perfect squares.

Feb 12-11:36 AM

What are perfect squares?

Perfect squares, or square numbers are those integers that have a square root that are also integers.

For example, 4 is a perfect square since its square root is 2. Consider a few more:
9 = 3x3
16 = 4x4
25 = 5x5
36=6x6

Feb 12-11:40 AM

Perfect squares allow us to make estimations as to square root values sans calculators.

Consider an example:

What is the square root of 37? (no calculator)

ANS: Well, since we know that the square root of 36 is 6 and the square root of 49 is 7, and 37 is between 36 and 49, we can safely assume that the square root of 37 must fall between 6 and 7, much closer to 6. Perhaps 6.1.
Check now what the calculator says.

Feb 12-11:47 AM

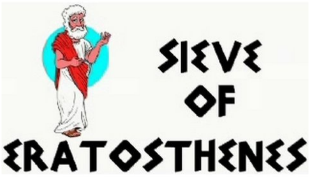
Now try the following handout, without a calculator. On the flip-side of the handout is a list of perfect squares to assist you.

Feb 12-11:53 AM

Sieve of Eratosthenes

Last week, we heard the news of the largest prime number ever found, a number over 17 million digits long! This was accomplished using software and the power of the internet. But how did ancient people manage to determine numbers that were prime? The first known test for primality is called the **Sieve of Eratosthenes**.

This process allows us to find all prime numbers up to a specified integer. It was developed by the Greek mathematician Eratosthenes, a little after Euclid determined there are an infinite number of primes.



Feb 11-11:29 AM

"Sift the Two's and Sift the Three's,
The Sieve of Eratosthenes.
When the multiples sublime,
The numbers that remain are Prime."

Anonymous

Just like a strainer gets rid of all the water and leaves only the cooked food, so too the Sieve of Eratosthenes sifts out all the composite numbers, leaving only the primes.

Feb 11-12:19 PM

PROOF that for any given x; $x^0=1$

First, let's revisit the rule for subtracting exponents:

Any time you have the same base in the numerator and denominator, you can rewrite the fraction by subtracting the exponent of the denominator's base from the exponent of the numerator's base.

Example:

$x^7/x^2 = x^{(7-2)} = x^5$.

What about x^a/x^a?

Well, that equals $x^{(a-a)}$ which simplifies to x^0. But remember, what is anything over itself?
Anything over itself is always 1. Therefore we have

$1 = x^a/x^a = x^0$.

Q.E.D.

Feb 6-9:37 AM

<u>For HW:</u>

Think about in this day and age, what possible information would be considered dangerous?

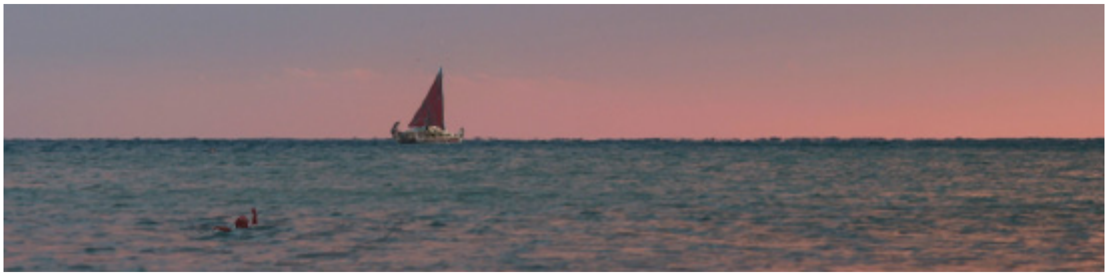
However, when answering this question, think specifically in regards to science and technology only.

Use your imagination, consider all the developments in recent years, what might governments not want the general public to know about?

Feb 12-11:41 AM

The Dangerous Ratio

Article by Brian Clegg



It's a stormy day on the sea off the coast of Greece. The date is around 520 BC. Fighting for his life, a man is heaved over the side of a boat and dropped into the open water to die. His name is Hippasus of Metapontum. His crime? Telling the world a mathematical secret. The secret of the dangerous ratio.

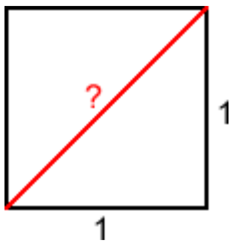
The murder of Hippasus is a matter of legend, but the secret was real, and certainly dangerous enough to the beliefs of those who knew about it.

It was a secret owned by the school of Pythagoras. These early Greek mathematicians (Pythagoras himself was born around 569 BC) were obsessed with the significance of whole numbers and their ratios. The Pythagorean's motto, carved above the entrance of the school, was "All is number".

The inner circle of the school, the mathematikoi, believed that the universe was built around the whole numbers. Each number from one to ten was given a very special significance. Odd numbers were thought to be male and even numbers female. Yet there was one number that the Pythagoreans found terrifying, the number that might have cost Hippasus his life for revealing its existence to the world.

The name Pythagoras these days is best remembered for a geometrical theorem, the one that tells us how to calculate the lengths of the sides of a right angled triangle, and it is from this theorem that the dangerous ratio emerges.

Imagine a simple square shape, each side 1 unit in length. How long is the square's diagonal?



This seemingly harmless question was the trigger for the Pythagoreans' disturbing discovery. The length of the square's diagonal is easy to work out. It forms the long side of a triangle with a right angle opposite, and two other sides of length 1 unit. Thanks to Pythagoras' theorem we (and the Greeks) know that we can work out the square of the length of the longest side of a right-angled triangle by adding together the squares of the other two sides. So we know the diagonal's length squared is $(1\times1)+(1\times1)=2$, making the length of the diagonal itself $\sqrt{2}$. The number which when multiplied by itself makes 2. But what is that number?

The square root of 2 isn't 1 because 1x1 is 1.

And it isn't 2, because 2x2 is 4.

It's something in between.

This wasn't a problem for the Pythagoreans. It was obviously a ratio of two whole numbers. They only had to figure out what that ratio was. At least that was the theory.

But after more and more frantic attempts, a horrible discovery was made. There is NO ratio that will produce $\sqrt{2}$ - it simply can't be done. It's what we now call an irrational number, not because it is illogical, but because it can't be represented as a ratio of whole numbers.

This was what sent the Pythagoreans into such a spin that they may have sacrificed poor Hippasus. If you believe that everything is constructed from whole numbers, it is a terrible a shock to discover that there is an everyday number, a 'real world' number like the diagonal of a square, that doesn't fit your picture of the world. It's a nightmare - and one from which the Pythagoreans would never really recover.

Sieve of Eratosthenes Worksheet

Name: ____

The Sieve of Eratosthenes is an ancient method for finding all primes numbers up to a specified number. It was created by Eratosthenes (275-194 B.C., Greece), an ancient Greek mathematician. Just as a sieve is a strainer for draining spaghetti, Eratosthenes's sieve drains out composite numbers and leaves prime numbers behind. The numbers from 1 to 100 are listed in the table below. We will use The Sieve of Eratosthenes to find all primes up to the number 100 by following the directions below.

Directions:

1. Cross out 1 since it is not prime.
2. Circle 2 because it is the smallest prime number. Cross out every multiple of 2.
3. Circle the next open number, 3. Now cross out every multiple of 3.
4. Circle the next open number, 5. Now cross out every multiple of 5.
5. Circle the next open number, 7. Now cross out every multiple of 7.
6. Continue this process until all numbers in the table have been circled or crossed out.

You have just circled all the prime numbers from 1 to 100!

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Questions:

1. How many prime numbers are there from 1 to 100?

2. List all prime numbers from 1 to 100.

3. Which number is the only even prime number?

4. An emirp (prime spelled backwards) is a prime that gives you a different prime when its digits are reversed. For example, 13 and 31 are emirps. List all emirps between 1 and 100.

Perfect squares chart

Number	Number Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625

Find each square root

1. $\sqrt{29}=$

2. $\sqrt{1}=$

3. $\sqrt{110}=$

4. $\sqrt{49}=$

5. $\sqrt{80}=$

6. $\sqrt{217}=$

7. $\sqrt{218}=$

8. $\sqrt{193}=$

9. $\sqrt{47}=$

10. $\sqrt{4}=$

11. $\sqrt{93}=$

12. $\sqrt{165}=$

13. $\sqrt{78}=$

14. $\sqrt{116}=$

15. $\sqrt{223}=$

16. $\sqrt{116}=$

17. $\sqrt{96}=$

18. $\sqrt{6}=$

19. $\sqrt{42}=$

Kuta Software - Infinite Geometry

Name_______________________

Simplifying Square Roots

Date________________ Period____

Simplify.

1) $\sqrt{96}$

2) $\sqrt{216}$

3) $\sqrt{98}$

4) $\sqrt{18}$

5) $\sqrt{72}$

6) $\sqrt{144}$

7) $\sqrt{45}$

8) $\sqrt{175}$

9) $\sqrt{343}$

10) $\sqrt{12}$

11) $10\sqrt{96}$

12) $9\sqrt{245}$

13) $7\sqrt{600}$

14) $5\sqrt{45}$

15) $5\sqrt{180}$

16) $3\sqrt{405}$

17) $2\sqrt{36}$

18) $9\sqrt{125}$

19) $8\sqrt{27}$

20) $12\sqrt{1764}$

21) $3\sqrt{900}$

22) $7\sqrt{2535}$

23) $11\sqrt{1215}$

24) $2\sqrt{200}$

Kuta Software - Infinite Geometry

Name________________________________

Simplifying Square Roots

Date________________ Period____

Simplify.

1) $\sqrt{96}$

$4\sqrt{6}$

2) $\sqrt{216}$

$6\sqrt{6}$

3) $\sqrt{98}$

$7\sqrt{2}$

4) $\sqrt{18}$

$3\sqrt{2}$

5) $\sqrt{72}$

$6\sqrt{2}$

6) $\sqrt{144}$

12

7) $\sqrt{45}$

$3\sqrt{5}$

8) $\sqrt{175}$

$5\sqrt{7}$

9) $\sqrt{343}$

$7\sqrt{7}$

10) $\sqrt{12}$

$2\sqrt{3}$

11) $10\sqrt{96}$

$40\sqrt{6}$

12) $9\sqrt{245}$

$63\sqrt{5}$

13) $7\sqrt{600}$

$70\sqrt{6}$

14) $5\sqrt{45}$

$15\sqrt{5}$

15) $5\sqrt{180}$

$30\sqrt{5}$

16) $3\sqrt{405}$

$27\sqrt{5}$

17) $2\sqrt{36}$

12

18) $9\sqrt{125}$

$45\sqrt{5}$

19) $8\sqrt{27}$

$24\sqrt{3}$

20) $12\sqrt{1764}$

504

21) $3\sqrt{900}$

90

22) $7\sqrt{2535}$

$91\sqrt{15}$

23) $11\sqrt{1215}$

$99\sqrt{15}$

24) $2\sqrt{200}$

$20\sqrt{2}$

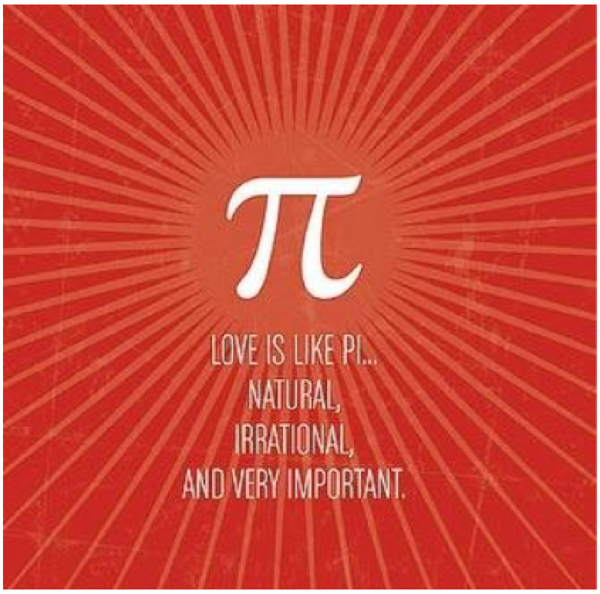
Lesson 7 Summary: February 14th

Euclid's 5 Postulates
Archimedes
Heron's Formula
Probability HW

Lesson Handouts

Heron's Formula worksheet

It is Valentine's Day so....



WARM UP:

<, >

1. What is the square root of 88? no calculator!

2. Is 200 a perfect square? Why or why not? What is its approximate square root value?

1<3U

3. Describe in your own words the process of using the Sieve of Eratosthenes.

4. What is the only number that fails in the proof $x^0 = 1$? Why does it fail?

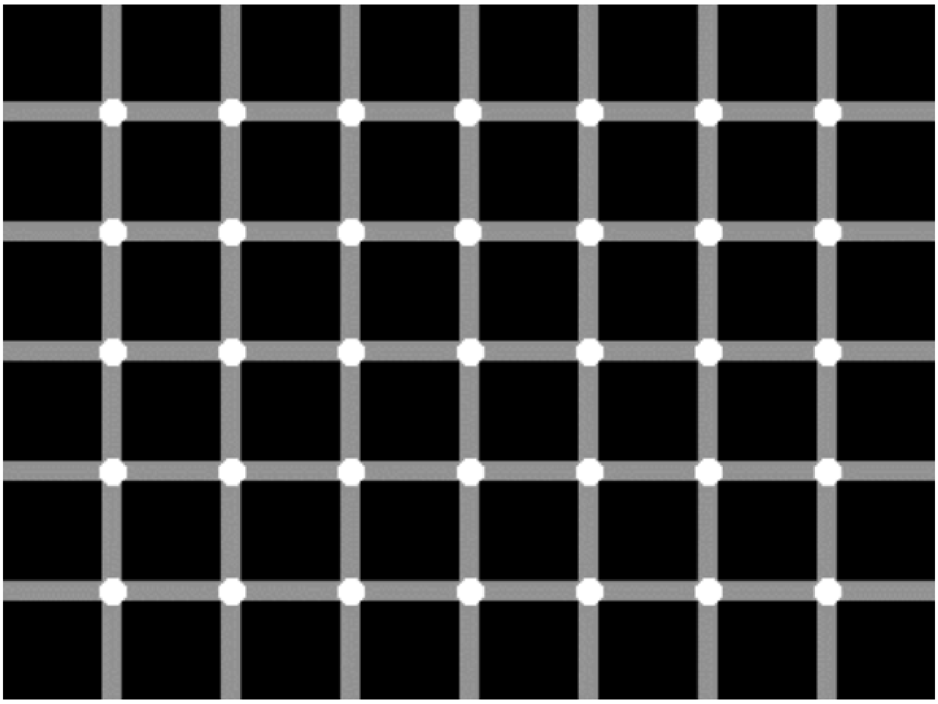
$\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

5. What is the probability of getting a odd prime number on a single roll of a die AND a heads on a single flip of a fair coin?

6. FUN: Solve for i (left side): 9x - 7i > 3(3x - 7u)

More Optical Illusion Fun:

There are only white circles at the intersections



Euclid's 5 Postulates

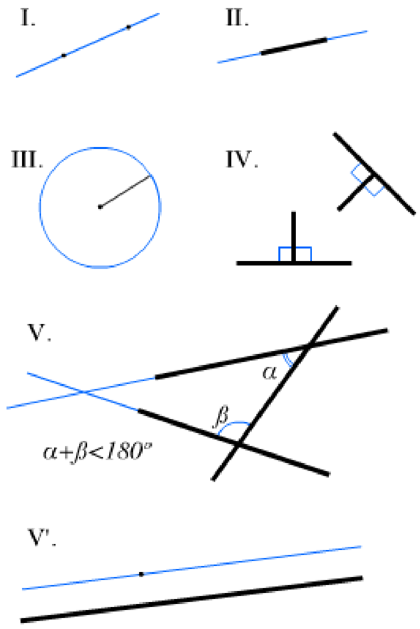
1. A straight line segment can be drawn joining any two points.

2. Any straight line segment can be extended indefinitely in a straight line.

3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.

4. All right angles are congruent.

5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the *parallel postulate*.



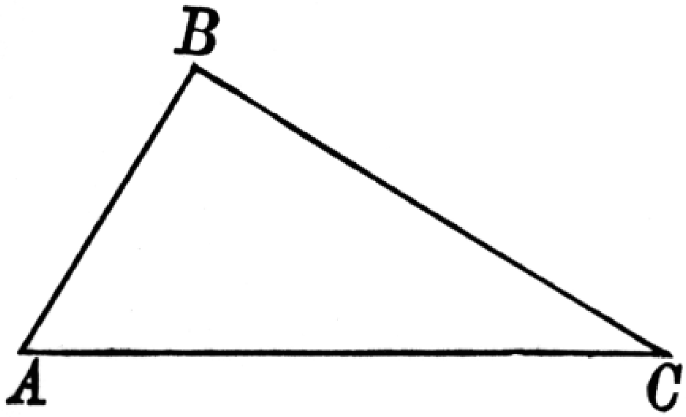
Euclid's fifth postulate cannot be proven as a theorem, although this was attempted by many people. Euclid himself used only the first four postulates ("absolute geometry") for the first 28 propositions of the Elements, but was forced to invoke the parallel postulate on the 29th.

In 1823, Janos Bolyai and Nicolai Lobachevsky independently realized that entirely self-consistent "non-Euclidean geometries" could be created in which the parallel postulate did not hold. (Gauss had also discovered but suppressed the existence of non-Euclidean geometries.)

Let's answer a few easy questions now about these 5 postulates, which were key ingredients in the development of geometry.

Euclidean Geometry Questions:

1. If you are told, "Figure ABC is a triangle," what information does that give you? *HINT: Try to draw the answer.*

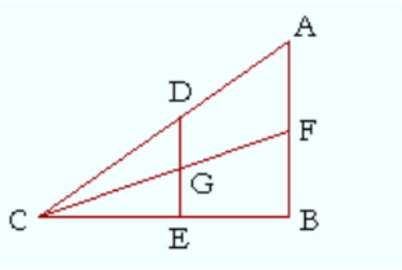


According to the definition of a triangle, you would know that AB, BC, and CA are straight lines.

2. Which Postulate guarantees that a triangle could actually exist?

ANS: Postulate 1, because it grants that we may draw a straight line betwee any two points.

3. Complete the following with the name of a straight line.



a) AB is equal to AF added to ___.

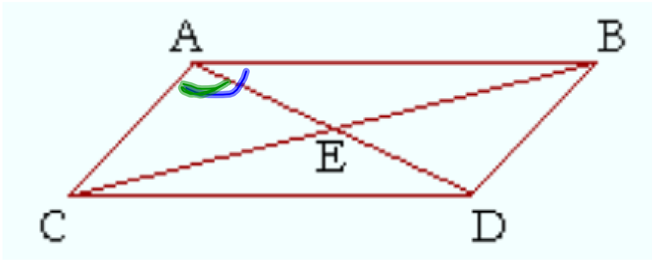
b) If GF is added to CG, then the whole is ___.

c) If DA is subtracted from CA, then what remains is ___.

d) If EC is subtracted from BC, then what remains is ___.

e) GE added to ___ is equal to DE.

4. Complete the following by naming an angle.



a) Angle CAB is equal to angles CAD, ___ together.

b) Angle ACD is equal to angles ECD, ___ together.

c) Angle EBD together with angle EBA is equal to angle ___.

d) If angle ACB is subtracted from angle ACD, then what remains is angle ___.

e) If angle ADB is subtracted from angle CDB, then what remains is angle ___.

Archimedes of Syracuse:

"Archimedes will be remembered when Aeschylus (a playwright) is forgotten, because languages die, and mathematical ideas do not. Immortality may be a silly word, but probably a mathematician has the best chances of whatever it may mean."
G. H. Hardy

As great as Euclid was, the Greek mathematician Archimedes perhaps even has a greater reputation. He lived from 287 BC - 212 BC. Archimedes was a true polymath (Renaissance Man). He was a physicist, he was an inventor, an engineer, an astronomer. He is considered by many as the leading scientist of Antiquity.

He gave the world the first rigorous accuracy of Pi, a value between 223/71 and 22/7. *Calculate what these two values are as decimal approximations.*

In mathematics, at the highest level, there is no Nobel Prize, as there is for Physics and other sciences. Instead, in math, we have the Fields Medal. As proof of Archimedes influence, his face adorns the Fields Medal.

The inscription around the head of Archimedes is a quote attributed to him which reads in Latin: "Transire suum pectus mundoque potiri"

Rise above oneself and grasp the world.

Heron's Formula

An important theorem in plane geometry, also known as Hero's formula. Given the lengths of the sides a, b, and c and the semiperimeter

$$s \equiv \frac{1}{2}(a+b+c)$$

of a triangle, Heron's formula gives the area Δ of the triangle as

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}\,.$$

Heron's formula is distinguished from other formulas for the area of a triangle, such as half the base times the height, by requiring no arbitrary choice of side as base or vertex as origin.

The formula is credited to Heron (or Hero) of Alexandria, and a proof can be found in his book, Metrica, written c. A.D. 60. **It has been suggested that Archimedes knew the formula over two centuries earlier.**

Since this could be called Archimedes' Formula, let's do some problems with this key geometric formula.

What is the area of a triangle whose sides are 12, 16 and 20 meters long?

ANS: 96 sq meters

What is the area of an equilateral triangle with all sides 6 inches in length?

area = $\sqrt{s(s-a)(s-b)(s-c)}$

area = $\sqrt{9(9-6)(9-6)(9-6)}$

area = $\sqrt{9(3)(3)(3)} = 9\sqrt{13} \approx 15.588$ square inches

HARDER: What equilateral triangle would have the same area as a triangle with sides 6, 8 and 10? $=24$

HINT: Equilateral Triangles have the formula: $Area = \frac{\sqrt{3}}{4}s^2$

1) $24 = \frac{\sqrt{3}}{4}s^2$

2) $96 = \sqrt{3}s^2$

3) $\frac{96}{\sqrt{3}} = s^2$

4) $\sqrt{\frac{96}{\sqrt{3}}} = s$

ANS: One where each side has a length of 7.44

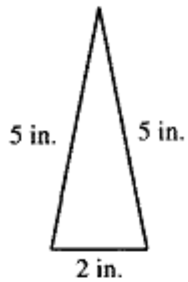
Probability: Expect the Unexpected

HW: Let's discuss a bit and try to answer this for Tuesday

"You discover two booths at a carnival. Each is tended by an honest man with a pair of covered coin shakers. In each shaker is a single coin, and you are allowed to bet upon the chance that both coins in that booth's shakers are heads after the man in the booth shakes them, does an inspection, and can tell you that at least one of the shakers contains a head. The difference is that the man in the first booth always looks inside both of his shakers, whereas the man in the second booth looks inside only one of the shakers. Where will you stand the best chance?"

HERON'S FORMULA Worksheet

1.



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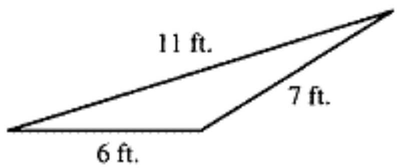

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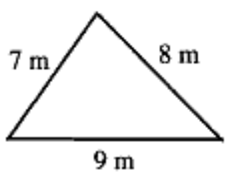

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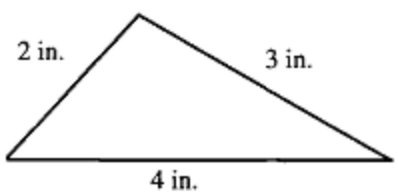

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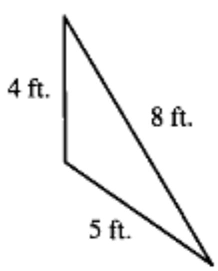

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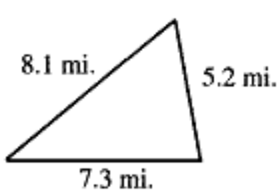


1. area = $\sqrt{24} \approx 4.899$ sq.in. **2.** area = $\sqrt{360} \approx 18.974$ sq.ft. **3.** area = $\sqrt{720} \approx 26.833$ sq.mi. **4.** area ≈ 2.905 sq.in. **5.** area ≈ 8.182 sq.ft. **6.** area ≈ 18.62 sq.mi.

Lesson 8 Summary: February 19th

The Nine Chapters on Mathematical Art
Distance = Rate x Time

Lesson Handouts

DRT Word Problems (Kuta)

The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.

- Henri Poincaré

WARM UP:

$\frac{A+B+C}{2} = S$

1. Use Heron's Formula (aka Archimedes' Formula) to find the area of an equilateral triangle with side of length 5.

$\frac{3x}{2} = S$

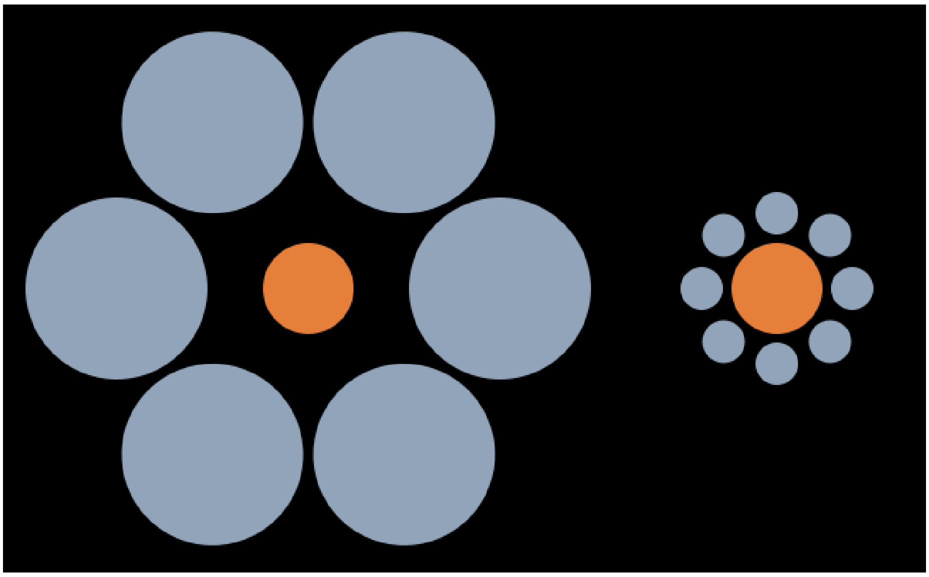
2. If you know that the value of "s" is 48 for an equilateral triangle, what are the lengths of the sides of the triangle?

3. What are Euclid's 5 postulates? (use your notes as needed)

4. On what Medal is Archimedes face adorned? What is the medal for?

5. If there is a 20% probability of snow and a 1% probability of having a winning lottery ticket, what is the chance that it will not snow and you will win the lottery?

More Optical Illusion Fun:



What you think is Dangerous

Direct governmental control (ie Patriot Act)

Indirect governmental control (ie farming and food supply chains controlled by large corporations controlled by the government)

Black Hat Hackers

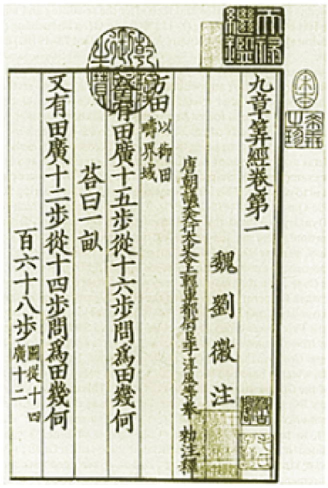
Any potential secrets that would disrupt our way of lives or send cultural shockwaves ie ET, Asteroids etc

HW: Probability: Expect the Unexpected

"You discover two booths at a carnival. Each is tended by an honest man with a pair of covered coin shakers. In each shaker is a single coin, and you are allowed to bet upon the chance that both coins in that booth's shakers are heads after the man in the booth shakes them, does an inspection, and can tell you that at least one of the shakers contains a head. The difference is that the man in the first booth always looks inside both of his shakers, whereas the man in the second booth looks inside only one of the shakers. Where will you stand the best chance?"

Probability HW Solution:

The key to solving many probability problems is to list out the possible scenarios.
So for our carnival problem, it helps to put ourselves into the shoes of the two men to make sense of the possibilities.
The first man looks at both shakers and says there is at least one head. This means that the possibilities are: HT, TH or HH.
The second man looks at one shaker and says there is at least one head. This means the possibilities are HT or HH. Why not TH? Well, he is looking directly at one of the shakers and sees a head, so either TH or HT has to be excluded.
Therefore there is a 1 in 3 chance you get the winning HH with the first man and a 1 in 2 chance that you get the winning HH with the second man.



The Jiuzhang suanshu or the Nine Chapters on Mathematical Art is the most famous ancient Chinese mathematical text. It was written during the period of the Han Dynasty, around 206 to 220 AD. It is not dissimilar to Euclid's Elements, although it was written about 600 years later. It is widely believed that Eastern and Western mathematics developed independently so it is interesting/useful because it sums up the mathematical knowledge of China at the time. We do not know the author and it contains 246 problems, categorized as:

1. Fangtian - Land Surveying. Area formulas, working with fractions, approximation of Pi.
2. Sumi - Millet and rice. Exchange of commodities at different rates; pricing, percentages & proportions.
3. Cuifen - Arithmetic & Geometric Progressions. Distribution of commodities and money at proportional rates.
4. Shaoguang - Square and cube roots; dimensions, area and volume of circle and sphere.
5. Shanggong - Volumes of solids of various shapes.
6. Junshu - Equitable taxation. More advanced problems on proportion.
7. Yingbuzu - Excess and deficit. Linear problems solved using the principle known later in the West as the rule of false position or trial and error.
8. Fangcheng - The rectangular array. Systems of linear equations, solved by a principle similar to Gaussian elimination.
9. Gougu - Base and altitude. Problems involving the principle known in the West as the Pythagorean theorem.

Let's now delve into a few of the specific problems found in The Nine Chapters on Mathematical Art, translated for you pleasure!

1. A good runner can go 100 paces while a poor runner covers 60 paces. The poor runner has covered a distance of 100 paces before the good runner sets off in pursuit. How many paces does it take the good runner before he catches up the poor runner.

2. (#26) A cistern is filled through five canals. Open the first canal and the cistern fills in 1/3 day; with the second, it fills in 1 day; with the third, in 2.5 days; with the fourth, in 3 days, and with the fifth in 5 days. If all the canals are opened, how long will it take to fill the cistern?

*ANS1: Several ways to set up, the idea is to set up an equation where the good runner distance equals the poor runner's distance. So 100x=60x + 100 (notice we add 100 to the poor runner since they have a 100 paces head start) We solve for x to get 2.5. However, we want to know the total distance so 2.5*100 =250 paces.*

ANS2:
Convert each canal into a unit rate, say per day. Then sum all the canals unit rates to get overall per day working together. Then 1/(previous result) to get portion of a day if all work together. So 3 + 1 + 1/2.5 + 1/3 + 1/5 = (45+15+7.5+5+3)/15 = 5.03
working together, they would fill the cistern 5 times over in a day, therefore 1/5.03 equals the time for 1 day or about 1/5 of a day

Name________________________________

Distance - Rate - Time Word Problems

Date________________ Period____

1) An aircraft carrier made a trip to Guam and back. The trip there took three hours and the trip back took four hours. It averaged 6 km/h on the return trip. Find the average speed of the trip there.

2) A passenger plane made a trip to Las Vegas and back. On the trip there it flew 432 mph and on the return trip it went 480 mph. How long did the trip there take if the return trip took nine hours?

3) A cattle train left Miami and traveled toward New York. 14 hours later a diesel train left traveling at 45 km/h in an effort to catch up to the cattle train. After traveling for four hours the diesel train finally caught up. What was the cattle train's average speed?

4) Jose left the White House and drove toward the recycling plant at an average speed of 40 km/h. Rob left some time later driving in the same direction at an average speed of 48 km/h. After driving for five hours Rob caught up with Jose. How long did Jose drive before Rob caught up?

5) A cargo plane flew to the maintenance facility and back. It took one hour less time to get there than it did to get back. The average speed on the trip there was 220 mph. The average speed on the way back was 200 mph. How many hours did the trip there take?

6) Kali left school and traveled toward her friend's house at an average speed of 40 km/h. Matt left one hour later and traveled in the opposite direction with an average speed of 50 km/h. Find the number of hours Matt needs to travel before they are 400 km apart.

7) Ryan left the science museum and drove south. Gabriella left three hours later driving 42 km/h faster in an effort to catch up to him. After two hours Gabriella finally caught up. Find Ryan's average speed.

8) A submarine left Hawaii two hours before an aircraft carrier. The vessels traveled in opposite directions. The aircraft carrier traveled at 25 mph for nine hours. After this time the vessels were 280 mi. apart. Find the submarine's speed.

9) Chelsea left the White House and traveled toward the capital at an average speed of 34 km/h. Jasmine left at the same time and traveled in the opposite direction with an average speed of 65 km/h. Find the number of hours Jasmine needs to travel before they are 59.4 km apart.

10) Jose left the airport and traveled toward the mountains. Kayla left 2.1 hours later traveling 35 mph faster in an effort to catch up to him. After 1.2 hours Kayla finally caught up. Find Jose's average speed.

Kuta Software - Infinite Algebra 1 Name________________________________

Distance - Rate - Time Word Problems

Date________________ Period____

1) An aircraft carrier made a trip to Guam and back. The trip there took three hours and the trip back took four hours. It averaged 6 km/h on the return trip. Find the average speed of the trip there.

8 km/h

2) A passenger plane made a trip to Las Vegas and back. On the trip there it flew 432 mph and on the return trip it went 480 mph. How long did the trip there take if the return trip took nine hours?

10 hours

3) A cattle train left Miami and traveled toward New York. 14 hours later a diesel train left traveling at 45 km/h in an effort to catch up to the cattle train. After traveling for four hours the diesel train finally caught up. What was the cattle train's average speed?

10 km/h

4) Jose left the White House and drove toward the recycling plant at an average speed of 40 km/h. Rob left some time later driving in the same direction at an average speed of 48 km/h. After driving for five hours Rob caught up with Jose. How long did Jose drive before Rob caught up?

6 hours

5) A cargo plane flew to the maintenance facility and back. It took one hour less time to get there than it did to get back. The average speed on the trip there was 220 mph. The average speed on the way back was 200 mph. How many hours did the trip there take?

10 hours

6) Kali left school and traveled toward her friend's house at an average speed of 40 km/h. Matt left one hour later and traveled in the opposite direction with an average speed of 50 km/h. Find the number of hours Matt needs to travel before they are 400 km apart.

4 hours

7) Ryan left the science museum and drove south. Gabriella left three hours later driving 42 km/h faster in an effort to catch up to him. After two hours Gabriella finally caught up. Find Ryan's average speed.

28 km/h

8) A submarine left Hawaii two hours before an aircraft carrier. The vessels traveled in opposite directions. The aircraft carrier traveled at 25 mph for nine hours. After this time the vessels were 280 mi. apart. Find the submarine's speed.

5 mph

9) Chelsea left the White House and traveled toward the capital at an average speed of 34 km/h. Jasmine left at the same time and traveled in the opposite direction with an average speed of 65 km/h. Find the number of hours Jasmine needs to travel before they are 59.4 km apart.

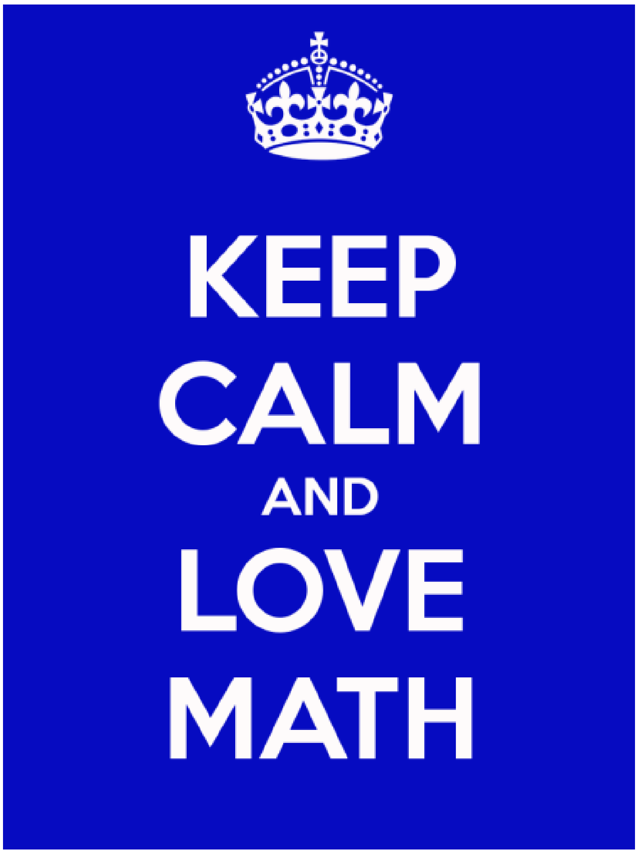
0.6 hours

10) Jose left the airport and traveled toward the mountains. Kayla left 2.1 hours later traveling 35 mph faster in an effort to catch up to him. After 1.2 hours Kayla finally caught up. Find Jose's average speed.

20 mph

Lesson 9 Summary: February 21st
Round 2: The Nine Chapters on Mathematical Art
Round 2: Distance = Rate x Time

Lesson Handouts
DRT Word Problems (Kuta)



WARM UP:

1. How many total questions are in the Nine Chapters on the Mathematical Art?

2. Using Heron's Formula, find the area of a triangle with dimensions of 3,4,5 (first pythagorean triple). Then use the general formula for the area of a triangle (A=.5BH) to confirm your result.

3. What is the formula that relates distance, time and rate?

4. What is the probability of getting the same numbers OR a sum of 6 on a single roll of a pair of dice?

5. If a 2lb bag of Jelly Belly jelly beans costs $18.49, and an ounce is 25 jelly beans worth, how many jelly beans are in the 2lb bag and what is the unit cost per bean? (16oz=lb)

Problem submitted by Robert:
Rufus T. Flypaper drives two miles to work every morning.
Very precise, he knows he must average 30mph to arrive on time.
One morning, a driver impedes him for the first mile, cutting his average to only 15mph. He quickly calculated his proper speed for the rest of his trip, to arrive on time. His car could do 120mph.
Could he arrive on time?

ANS:
D=RT
2=30T so T = 1/15 or 4 minutes. (This is Mr. Flypapers normal traveling time)
If the first mile is at 15mph, then he has traveled 1=15T so T =1/15 or 4 minutes. Therefore, he would need a teleportation device to ensure arriving on time, since at the halfway point (1 mile traveled, one mile to go) he has already spent 4 minutes, which is the time he has allotted to ensure he arrives on time.

More Optical Illusion Fun:

What is the name of the artist that created this work?

Let's spend a little time and go over all the answers to the worksheet from Tuesday.

$$480 \cdot 9 = 4320$$

$$RT = D, \quad T = \frac{D}{R}$$

$$\frac{4320}{432}$$

$$\frac{180}{18} = 10 \qquad \frac{D}{R} = T$$

$$RT_1 = RT_2$$

x , $x+1$

$$220x = 200(x+1)$$

$$220x = 200x + 200$$

$$20x = 200$$

$$x = 10$$

$$R \cdot T_1 + R_2 \cdot T_2 = D$$

$$40x + 50(x-1) = 400$$

$$40x + 50x - 50 = 400$$

$$90x = 450$$

$$x = 5$$

7) $R_1 T_1 = R_2 \cdot T_2$

$5X = 2(x+42)$

$5X = 2x + 84$

$3X = 84$

$X = 28$

8) $R_1 T_1 + R_2 T_2 = D$

$25 \cdot 9 + 11 \cdot X = 280$

$11X = 55$

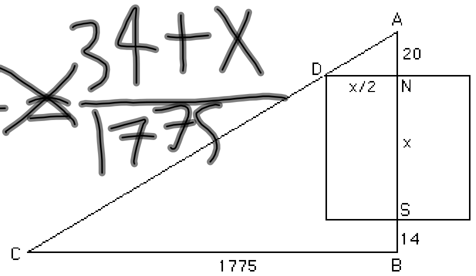
$X = 5$

Let's now delve into another problem found in The Nine Chapters on Mathematical Art, translated for your pleasure!

20. There is a square town of unknown dimensions. There is a gate in the middle of each side. Twenty paces outside the North Gate is a tree. If one leaves the town by the South Gate, walks 14 paces due south, then walks due west for 1775 paces, the tree will just come into view. What are the dimensions of the town?

HINT1: DRAW DRAW DRAW.
HINT2: Use the notion of similar triangles. That is, the ratio of two sides of one triangle is equal to the ratio of the two sides of a similar triangle
HINT3:

$$\frac{20}{\frac{x}{2}} = \frac{34+x}{1775}$$



HINT4: Use Quadratic Formula

$$\frac{AN}{DN} = \frac{AB}{CB}$$

ANS:
Now triangles AND and ABC are similar so AN/ND = AB/BC giving:
20/(x/2) = (20 + x + 14)/1775.
Then x2 + x(20 + 14) = 2 (20×1775), or x2 + 34x = 71000.
The side of the town is 250 paces.

$$35500 = (34+x)\frac{x}{2}$$

$$35500 = 17x + \frac{x^2}{2}$$

$$71000 = 34x + x^2$$

$$0 = x^2 + 34x - 71000$$

$$a = 1$$

$$b = 34$$

$$c = -71000$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-34 \pm \sqrt{1156 - (-284{,}000)}}{2}$$

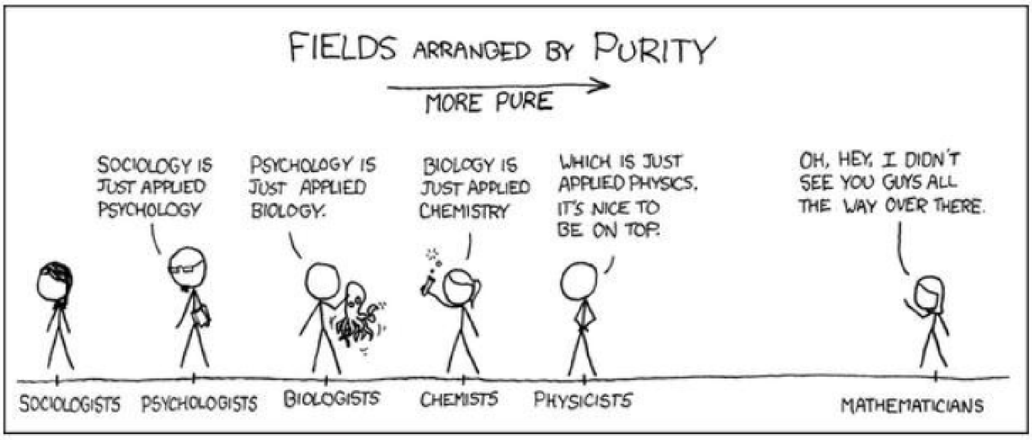
$$\frac{-34 \pm 534}{2} = \boxed{250}$$

HW Assignment:

Pick any number you like and do something with it. You could write a report on its properties, or make a powerpoint or draw something, be creative.

Lesson 10 Summary: February 26th
Euclidean Algorithm
Ptolemy & the Almagest
Hypatia

Lesson Handouts
Death of Hypatia (Cliff Pickover, The Math Book)
Basic Astronomy Quiz



Let's be honest, everything is just applied mathematics.

WARM UP:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Solve $x^2 + 3x - 4 = 0$ using the quadratic formula.

2. If the solutions to question one are in terms of distance, which solution can you discard and why?

3. Solve $9x^2 + 12x + 4 = 0$ using the quadratic formula.

4. Superheroes Flash and Flash Jr. leave the same location and run in opposite directions. Flash Jr. runs 1 mile per second (mps) and Flash runs 2 mps. How far apart are they in miles after 1 hour?

5. If the circumference of the earth is 24,000 miles, what percentage around the earth do they cover in one hour?

6. How many centuries are in 250,000 days? (Be accurate to within one year.)

$$\frac{-12 \pm \sqrt{0}}{18} =$$

$$\frac{-12}{18} = \frac{-2}{3}$$

$$9x^2 + 12x + 4 = 0$$

$$(3x+2)(3x+2) = \frac{0}{3x+2}$$

$$3x + 2 = 0$$

$$\frac{-3 \pm \sqrt{9-(4 \cdot 1 \cdot -4)}}{2} =$$

$$\frac{-3 \pm \sqrt{9 - -16}}{2} =$$

$$\frac{-3 \pm \sqrt{25}}{2}$$

$$\frac{-3 \pm 5}{2}$$

$$(1, -4)$$

More Optical Illusion Fun:

Remember, remember, remember: Distance = Rate x Time

Susan (with her pooch in tow) can row a boat 10 kilometers per hour in still water. In a river where the current is 5 kilometers per hour, it takes her 4 hours longer to row a given distance upstream than to travel the same distance downstream. Find how long it takes her to row upstream, how long to row downstream, and how many kilometers she rows.

	R	x T	= D
STILL	10	3	
D	15	x=2	30
U	5	x+4	30

$15x = 5x + 20$

$10x = 20$

$x = 2$

In the section of The Nine Chapters on Mathematical Art, there is a process revealed which is known in the Western world as the Euclidean Algorithm. It allows us to find the greatest number that is a divisor of two given integers.

Formal description of the Euclidean algorithm

Input Two positive integers, a and b.

Output The greatest common divisor, g, of a and b.

Internal computation
If a<b, exchange a and b.
Divide a by b and get the remainder, r. If r=0, you are done, b is the GCD of a and b.
Otherwise, replace a by b and replace b by r. Return to the previous step.

Now try 45 and 210 using the algorithm.

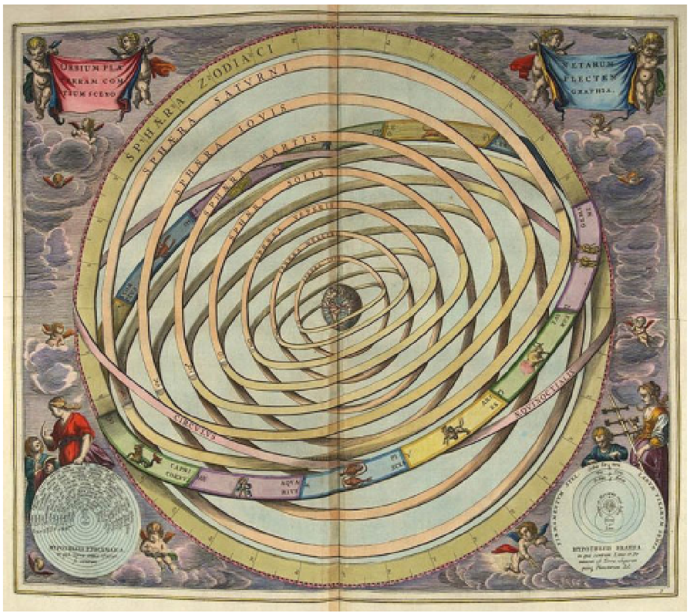
ANS: Divide 210 by 45, and get the result 4 with remainder 30, so $210=4\cdot45+30$.
Divide 45 by 30, and get the result 1 with remainder 15, so $45=1\cdot30+15$.
Divide 30 by 15, and get the result 2 with remainder 0, so $30=2\cdot15+0$.
The greatest common divisor of 210 and 45 is 15.

What is the difference between astrology and astronomy? Are they the same thing?

The primary goal of astronomy is to understand the physics of the universe. Astrologers use astronomical calculations for the positions of celestial bodies along the ecliptic and attempt to correlate celestial events (astrological aspects, sign positions) with earthly events and human affairs. Astronomers consistently use the scientific method, naturalistic presuppositions and **abstract mathematical reasoning** to investigate or explain phenomena in the universe. Astrologers use **mystical or religious reasoning** as well as traditional folklore, symbolism and superstition blended with mathematical predictions to explain phenomena in the universe. The scientific method is not consistently used by astrologers.

Astrologers believe that the position of the stars and planets determine an individual's personality and future. Astronomers study the actual stars and planets, but have found no evidence supporting astrological theories. Psychologists study personality, and while there are many theories of personality, no mainstream theories in that field are based on astrology.

In a similar way that Euclid's Elements helped to create the foundation of Geometry, the 13 book **Almagest** had a similar impact on Astronomy. Written by **Ptolemy of Alexandria**, this tome covers almost everything that was known about astronomy at the time.



One of the great astronomers and mathematicians around this era was also one of the first documented women mathematicians. Her name was **Hypatia**. She was a teacher in philosophy, mathematics and astronomy. She is likely to have made updates to Euclid's Elements and Ptolemy's Almagest. Unfortunately she was likely murdered by a Christian mob. It also has been argued by historians that her death marked a sharp downturn in the intellectual culture of the time.

There was a woman at Alexandria named Hypatia, daughter of the philosopher Theon, who made such attainments in literature and science, as to far surpass all the philosophers of her own time. Having succeeded to the school of Plato and Plotinus, she explained the principles of philosophy to her auditors, many of whom came from a distance to receive her instructions. On account of the self-possession and ease of manner which she had acquired in consequence of the cultivation of her mind, she not infrequently appeared in public in the presence of the magistrates. Neither did she feel abashed in going to an assembly of men. For all men on account of her extraordinary dignity and virtue admired her the more."

—Socrates Scholasticus, Ecclesiastical History

$$\frac{x^3}{x^1} = x^2$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^3)^2 = x^6$$

$$(x^A)^B = x^{AB}$$

$$x^3 \cdot x^2 = x^5$$

$$x^a \cdot x^b = x^{a+b}$$

$$x^5 \cdot x^3 \cdot x^{-6} =$$

$$x^{-7} =$$

$$x^0 = 1$$

$$x^{-a} = \frac{1}{x^a}$$

The Death of Hypatia

Hypatia of Alexandria (c. 370–c. 415)

Hypatia of Alexandria was martyred by being torn to shreds by a Christian mob, partly because she did not adhere to strict Christian principles. She considered herself a neo-Platonist, a pagan, and a follower of Pythagorean ideas. Interestingly, Hypatia is the first woman mathematician in the history of humanity of whom we have reasonably secure and detailed knowledge. She was said to be physically attractive and determinedly celibate. When asked why she was obsessed with mathematics and would not marry, she replied that she was wedded to the truth.

Hypatia's works include commentaries on **Diophantus's *Arithmetica***. In one of her mathematical problems for her students, she asked them for the integer solution of the pair of simultaneous equations: $x - y = a$ and $x^2 - y^2 = (x - y) + b$, where a and b are known. Can you find any integer values for x, y, a, and b that make both of these formulas true?

The Christians were her strongest philosophical rivals, and they officially discouraged her Platonic assertions about the nature of God and the afterlife. On a warm March day in A.D. 414, a crowd of Christian zealots seized her, stripped her, and proceeded to scrape her flesh from her bones using sharp shells. Next, they cut up her body and burned the pieces. Like some victims of religious terrorism today, she may have been seized merely because she was a famous person on the other side of the religious divide. It was not until after the Renaissance that another woman, Maria Agnesi, made her name as a famous mathematician.

Hypatia's death triggered the departure of many scholars from Alexandria and, in many ways, marked the end of centuries of Greek progress in mathematics. During the European Dark Ages, Arabs and Hindus were the ones to play the leading roles in fostering the progress of mathematics.

SEE ALSO Pythagoras Founds Mathematical Brotherhood (c. 530 B.C.), Diophantus's *Arithmetica* (250), Agnesi's *Instituzioni Analitiche* (1748), and The Doctorate of Kovalevskaya (1874).

In 1885, British painter Charles William Mitchell depicted Hypatia moments before her death at the hands of a Christian mob that stripped her and slaughtered her in a church. According to some reports, she was flayed with sharp objects and then burned alive.

In the following we will show how to solve simple problems in astronomy. One of the most common type of problem in astronomy (and everyday life, for that matter) is the distance - velocity - time problem. 1. For example, suppose you are driving cross-country at an average speed of 60mph. If you drive for 12 hours how far will you get?

A).. 5 miles
B).. 720 miles
C).. 72 miles
D).. 7200 miles
E).. 500 miles

2. How did you solve that problem?

A)... Multiplied the distance times the time.
B)... Multiplied the speed by the time.
C)... Divided the speed by the time.
D)... Divided the time by the speed.
E)... Added the speed to the time.

Notice that you used the basic relation that Distance = speed x time, or mathematically, D=Vt, where D is the distance you traveled, V is your average speed (more technically the velocity), and t is the time you traveled.

Now let's do a variation on that problem.

3. Suppose you are driving from Boston to New York. If you travel at an average speed of 50 mph and the distance is 250 miles, how long will the trip take?

A)... 300 minutes.
B)... 200 minutes.
C)... 5 hours.
D)... 5 minutes.
E)... 50 hours.

Notice that you again used the basic relation D=Vt, but now in the form time = distance/speed, or mathematically, t=D/V.

Let's now do an astronomical version of this problem.

P 4. The space shuttle travels around the Earth in a circular orbit whose circumference is about 25,000 miles. The shuttle's orbital speed is about 17,000 miles/hr. About how long does it take the shuttle to complete one orbit around the Earth? (Notice this is exactly like the driving problem above.)

A)... 1 hour
B)... 1.5 minutes
C)... 0.6 hours
D)... 1.5 hours
E)... 15 hours

Notice that in this problem none of the answers you chose from were the answer you found mathematically. Rather, you were asked to choose from rounded off answers. Such rounding off is very important in writing out answers. Generally 3 digits is accurate enough for our purposes.

For example,

5. round off 2.7248283...to three figures.

A)... 2.72
B)... 2.73
C)... 2.71
D)... 2.74
E)... 2.70

Let's now do another astronomical distance/velocity time problem.

6. A radar pulse is sent from the Earth to an asteroid, bounces off and returns to Earth. The radar pulse travels at the speed of light (300,000 km/sec) and takes 150 seconds to travel out and return. How far away is the asteroid? (Notice that this is exactly like

problem 1 above.)

A)... 2250 km
B)... 4500 km
C)... 45,000 km
D)... $4.5x10^7$ km
E)... $2.25 x10^7$ km

To better comprehend how much time the pulse takes, let's express the 150 seconds in minutes.

7. 150 seconds is about how many minutes? (As a quick check on your answer, will the number of minutes be more or less than the number of seconds?)

A)... 2.0
B)... 2.5
C)... 25
D)... 0.25
E)... 0.4

You will often have to make conversions of this sort in problems. For example, suppose we ask the following: How many years does it take a ray of light to travel from a star $12x10^{13}$ km from Earth to our planet? Recall the speed of light is $3x10^5$ km/sec.

In this problem you will get an answer in seconds, but you are asked for the answer in years. Thus, you need the number seconds in a year. You can figure that out by multiplying the number of seconds in a minute by the number of minutes in an hour by the number of hours in a day by the number of days in a year. In doing all that you will find that the number of seconds in a year is about 3×10^7.

8. Given this conversion factor, what is the answer to the above problem?

A)... About 1 year
B)... About 3 years
C)... About 13 years
D)... About 13 million years
E)... About 1.3 years

If you had trouble getting this answer, check your powers-of-ten math. For example,

9. what is 10^{13} divided by 10^5?

A)... 10^8
B)... $10^{2.6}$
C)... 10^{18}
D)... 8
E)... 2.6

Calculate Results Reset

Your Answers	Your answer to the question was
Q1:	Q1:
Q2:	Q2:
Q3:	Q3:
Q4:	Q4:
Q5:	Q5:
Q6:	Q6:
Q7:	Q7:
Q8:	Q8:
Q9:	Q9:

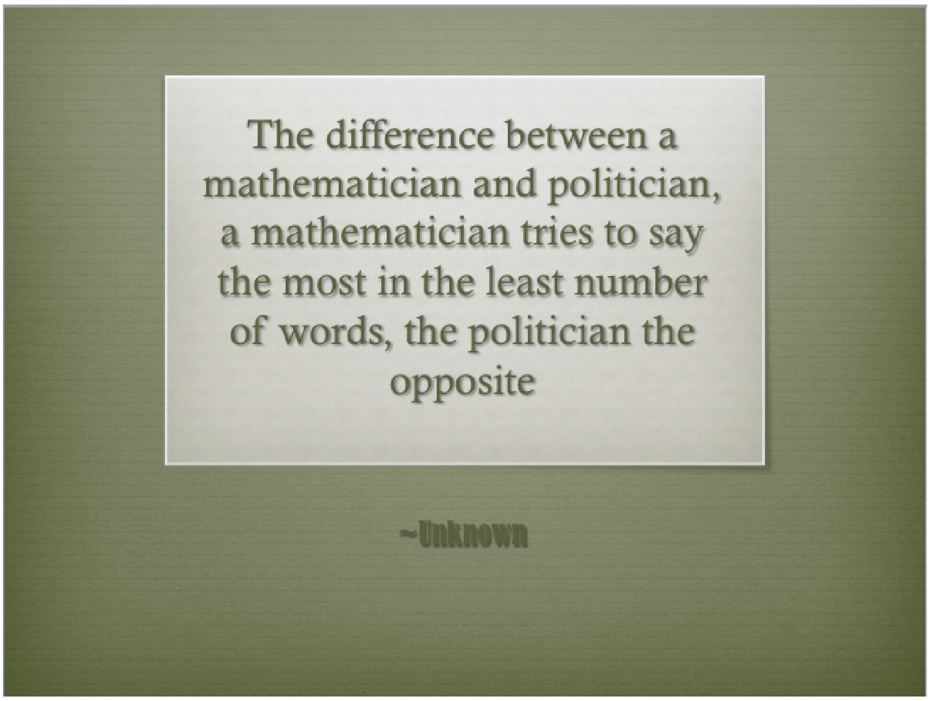
Lesson 11 Summary: February 28th

Zero

Rules of Exponents

Lesson Handouts

Properties and More Properties of Exponents worksheets (Kuta)



WARM UP:

1. What is the Almagest? Why is it important?

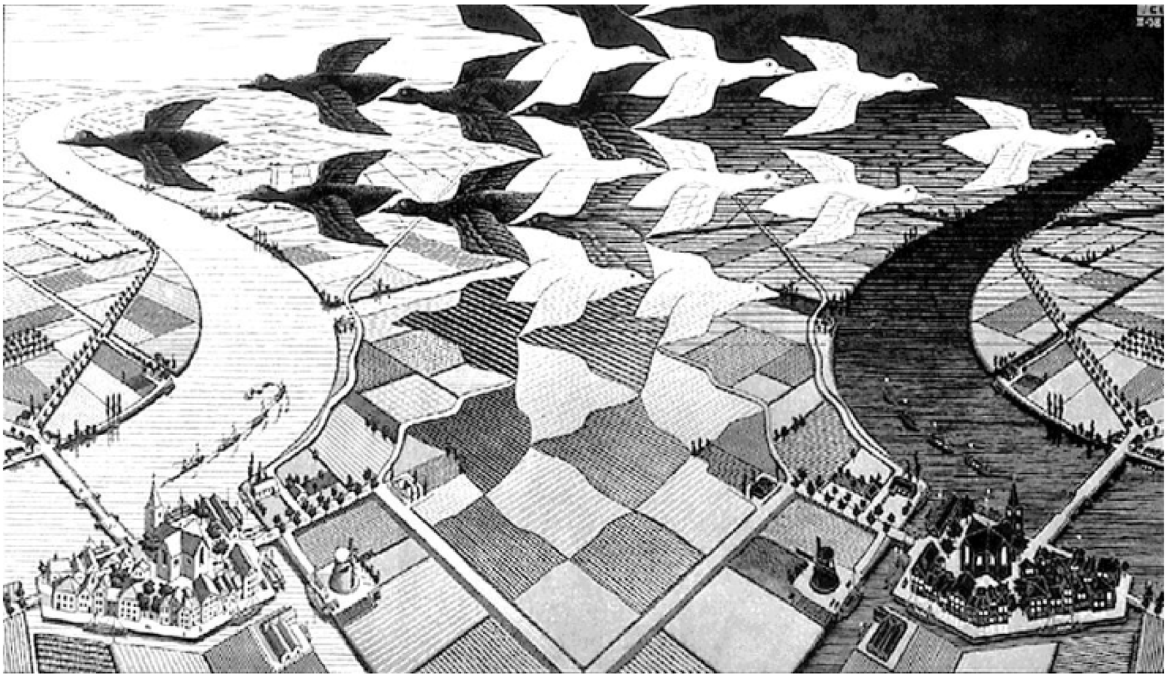
2. What is more reliable, astronomy or astrology? Why?

3. If your pet turtle Tommy is exactly 100 years old, how many hours old is Tommy? How many hours old are you? If you live to be 100, how many hours left do you have?

4. If light travels at 186,282.397 miles per second, and the distance that planet earth (aka home) is to the sun (aka an average star) is 93,000,000 miles away, how long does it take for sunlight to reach earth? Give your answer in terms of minutes.

5. There are 10 marbles in a bag: 3 red, 2 blue and 5 green. What are the chances you do not get a red AND get a heads on a single coin flip AND a prime number on a single die roll?

Visual Fun for the Day:



What formula do you need here?

Suppose two sisters live 240 miles apart. One sister has three young children who are planning to visit their aunt for a week. To prevent driving so far, the sisters agree to leave at the same time, drive toward each other, and meet somewhere along the route. The sister with the three children tends to drive carefully and obey the speed limit. Her average rate of speed is 70 mph. The other sister drives too fast, and her average rate of speed is 80 mph. How long will it take the two sisters to meet each other to transfer the children? Give your answer in minutes.

$R_1 \cdot T_1 + R_2 \cdot T = 240$

$70t + 80T = 240$

$150T = 240$

$t = 1.6$ → 96 MINS

Read the following essays on Hypatia and Zero and then answer the following questions to hand in:

1. If Hypatia was alive today, would her scientific challenges to the Christian church end in the same way? Can religion and science coexist?

2. Think of and list as many examples as you can where you might see the number zero in modern society. (in any context)

Since we have reached the point in history where some of the notions of astronomy are being figured out (Ptolemy's Almagest), we will focus on one of the key math skills you need for a lot of astronomy calculations, something we started discussing yesterday, namely the **rules of exponents**.

Why do you think that exponents are so prevalent in astronomy?

The Basic Rules of Exponents

Rule #1 – Product Rule: $x^m * x^n = x^{(m+n)}$

Rule #2 – Power Rule: $(x^m)^n = x^{(m*n)}$

Rule #3 – Product Rule-2 Bases: $x^m * y^n = x^m * y^n$

Rule #4 – Power Rule-2 Bases: $(x^m * y^n)^z = x^{(m*z)} * y^{(n*z)}$

$(x^m / y^n)^z = x^{(m*z)} / y^{(n*z)}$

Rule #5 – Negative Rule: $x^{-m} = 1/(x^m)$

Rule #6 – Quotient Rule: $x^m / x^n = x^{(m-n)}$

Rule #7 – Distributive Rule:

Exponents Distribute across multiplication and division

Exponents DO NOT Distribute across addition and subtraction

$(2x)^8 = 2^8 x^8$

$2x^8$

$$1x^{-12} \cdot 2x^{4} = 2x^{-8}$$

$$= \frac{2}{x^{8}}$$

$$\frac{24x^{5}y^{8}}{3x^{-3}y^{2}} = 8x^{8}y^{6}$$

$$\frac{6x^1y^6}{3x^2y^4} = 2x^{-1}y^2 = \frac{2y^2}{x}$$

$$\frac{x^6y^3}{4x^2} = \frac{x^2y^3}{4}$$

$$\frac{3x^2y^2}{8x^1y^1} = \frac{3x \cdot y}{8}$$

$$1\left(\frac{1^1}{1}\right)^1$$

9) $\frac{x}{4}$

10) $\frac{2m^{-4}}{8m^{-12}} =$

$$\frac{1 \cdot m^8}{4}$$

11)

$$\frac{2^{-1} m^{-2}}{m^2} =$$

 Name________________________

More Properties of Exponents

Date________________ Period____

Simplify. Your answer should contain only positive exponents.

1) $(x^{-2}x^{-3})^4$

2) $(x^4)^{-3} \cdot 2x^4$

3) $(n^3)^3 \cdot 2n^{-1}$

4) $(2v)^2 \cdot 2v^2$

5) $\dfrac{2x^2y^4 \cdot 4x^2y^4 \cdot 3x}{3x^{-3}y^2}$

6) $\dfrac{2y^3 \cdot 3xy^3}{3x^2y^4}$

7) $\dfrac{x^3y^3 \cdot x^3}{4x^2}$

8) $\dfrac{3x^2y^2}{2x^{-1} \cdot 4yx^2}$

9) $\dfrac{x}{(2x^0)^2}$

10) $\dfrac{2m^{-4}}{(2m^{-4})^3}$

11) $\dfrac{(2m^2)^{-1}}{m^2}$

12) $\dfrac{2x^3}{(x^{-1})^3}$

13) $(a^{-3}b^{-3})^0$

14) $x^4y^3 \cdot (2y^2)^0$

15) $ba^4 \cdot (2ba^4)^{-3}$

16) $(2x^0y^2)^{-3} \cdot 2yx^3$

17) $\dfrac{2k^3 \cdot k^2}{k^{-3}}$

18) $\dfrac{(x^{-3})^4 x^4}{2x^{-3}}$

19) $\dfrac{(2x)^{-4}}{x^{-1} \cdot x}$

20) $\dfrac{(2x^3z^2)^3}{x^3y^4z^2 \cdot x^{-4}z^3}$

21) $\dfrac{(2pm^{-1}q^0)^{-4} \cdot 2m^{-1}p^3}{2pq^2}$

22) $\dfrac{(2hj^2k^{-2} \cdot h^4j^{-1}k^4)^0}{2h^{-3}j^{-4}k^{-2}}$

Kuta Software - Infinite Algebra 1 Name________________________

More Properties of Exponents

Date________________ Period____

Simplify. Your answer should contain only positive exponents.

1) $\left(x^{-2}x^{-3}\right)^{4}$

$\frac{1}{x^{20}}$

2) $\left(x^{4}\right)^{-3} \cdot 2x^{4}$

$\frac{2}{x^{8}}$

3) $\left(n^{3}\right)^{3} \cdot 2n^{-1}$

$2n^{8}$

4) $(2v)^{2} \cdot 2v^{2}$

$8v^{4}$

5) $\frac{2x^{2}y^{4} \cdot 4x^{2}y^{4} \cdot 3x}{3x^{-3}y^{2}}$

$8x^{8}y^{6}$

6) $\frac{2y^{3} \cdot 3xy^{3}}{3x^{2}y^{4}}$

$\frac{2y^{2}}{x}$

7) $\frac{x^{3}y^{3} \cdot x^{3}}{4x^{2}}$

$\frac{x^{4}y^{3}}{4}$

8) $\frac{3x^{2}y^{2}}{2x^{-1} \cdot 4yx^{2}}$

$\frac{3xy}{8}$

9) $\frac{x}{\left(2x^{0}\right)^{2}}$

$\frac{x}{4}$

10) $\frac{2m^{-4}}{\left(2m^{-4}\right)^{3}}$

$\frac{m^{8}}{4}$

11) $\dfrac{(2m^2)^{-1}}{m^2}$

$\dfrac{1}{2m^4}$

12) $\dfrac{2x^3}{(x^{-1})^3}$

$2x^6$

13) $(a^{-3}b^{-3})^0$

1

14) $x^4y^3 \cdot (2y^2)^0$

x^4y^3

15) $ba^4 \cdot (2ba^4)^{-3}$

$\dfrac{1}{8b^2a^8}$

16) $(2x^0y^2)^{-3} \cdot 2yx^3$

$\dfrac{x^3}{4y^5}$

17) $\dfrac{2k^3 \cdot k^2}{k^{-3}}$

$2k^8$

18) $\dfrac{(x^{-3})^4x^4}{2x^{-3}}$

$\dfrac{1}{2x^5}$

19) $\dfrac{(2x)^{-4}}{x^{-1} \cdot x}$

$\dfrac{1}{16x^4}$

20) $\dfrac{(2x^3z^2)^3}{x^3y^4z^2 \cdot x^{-4}z^3}$

$\dfrac{8x^{10}z}{y^4}$

21) $\dfrac{(2pm^{-1}q^0)^{-4} \cdot 2m^{-1}p^3}{2pq^2}$

$\dfrac{m^3}{16p^2q^2}$

22) $\dfrac{(2hj^2k^{-2} \cdot h^4j^{-1}k^4)^0}{2h^{-3}j^{-4}k^{-2}}$

$\dfrac{h^3j^4k^2}{2}$

Lesson 12 Summary: March 5th

Round 2: Exponents
Response to Video on Museum of Math
Algebra Khawrizmi
Ganita

Lesson Handouts

Drake Equation

Class Update:

Next week we will have our midterm. This will be cumulative and will be worth 25% of your final grade. Please consult syllabus for more information. We will have the midterm on Tuesday so you can know you overall grade going into spring break.

We will spend part of class this Thursday going over what you will need to know.

Feb 27-9:48 AM

WARM UP:

1. Simplify: $\frac{(2x^3)(8x^5)}{4x^6} = \frac{16x^8}{4x^6}$

2. x^0= ? 1

3. Simplify 5^{-3} $\frac{1}{5^3}$ $\quad \frac{1}{5^{-3}} = 5^3$

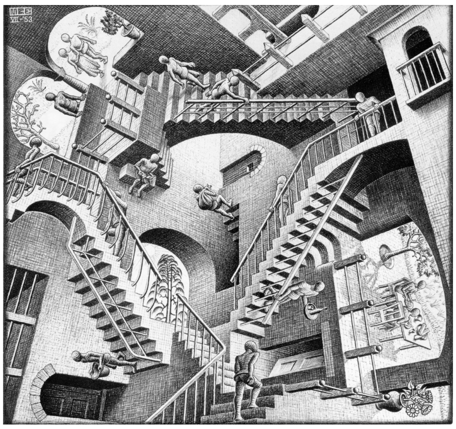
4. Convert your answer from #3 into a decimal. .008

5. If your answer to #4 represents the percentage of stock you own of company x, and company x is worth 350 million dollars, what is your stake in the company?

6. What is the square root of -25?

Feb 6-9:21 AM

Visual Fun for the Day:



Feb 11-12:17 PM

Let's finish the exponent worksheet we worked on last week that you were expected to complete for homework.

Mar 5-12:14 PM

$b^{1}a^{4} \cdot 2^{-3} b^{-3} a^{-12} =$

$b^{-2} a^{-8} 2^{-3} =$

$\frac{1}{8b^{2}a^{8}}$

Mar 5-1:08 PM

16)

$2^{-3} y^{-6} \cdot 2^{1} y^{1} x^{3} =$

$2^{-2} y^{-5} x^{3} =$

$\frac{x^{3}}{4y^{5}}$

Mar 5-1:13 PM

17) $2k^{8}$

18) $\frac{x^{-12}x^{4}}{2x^{-3}} = \frac{x^{-8--3}}{2} =$

$\frac{x^{-5}}{2} = \frac{1}{2x^{5}}$

Mar 5-1:15 PM

Watch this recent video and then write up a response/reflection to turn in.

http://www.cbsnews.com/video/watch/?id=50142058n

Mar 5-12:15 PM

"Watch your thoughts, for they become words.
Watch your words, for they become actions.
Watch your actions, for they become habits.
Watch your habits, for they become character.
Watch your character, for it becomes your destiny."

Mar 5-1:32 PM

ALGEBRA

A word that unfortunately conjures up dread and anxiety for too many people.

It comes from the word *al-jabr*, which is one of the two operations used to solve quadratic equations found in the book Al-Kitāb al-mukhtaṣar fī hīsāb al-ğabr wa'l-muqābala or in English, "The Compendious Book on Calculation by Completion and Balancing".

This is a mathematical book written in Arabic in approximately AD 820 in modern day Baghdad by the Persian mathematician Muhammad ibn Mūsā al-Khwārizmī. He is often considered the "father of algebra".

Mar 5-10:41 AM

Math, more than any other subject, builds on itself. This means that concepts you learn in one class appear in another class.
This might be why people start to resent math, if they do poorly in one class, they are likely to do poorly in the next one. For example, in order to be a great calculus student, it is critical that you are first a master of algebra since there is a great deal of algebra you are expected to know in Calculus.

With the notion of the sequential nature of mathematics in mind, let's work through some problems to ensure we understand some critical **algebraic concepts**!

Mar 5-11:48 AM

1) $6 = \frac{a}{4} + 2$

$4 = \frac{a}{4}$

$16 = A$

$(4p-1)^2 =$

$(4p-1)(4p-1) =$

$16p^2 - 4p - 4p + 1 =$

$16p^2 - 8p + 1$

Mar 5-1:45 PM

Around the same time as Muhammad ibn Mūsā al-Khwārizmī work comes the mathematical text Ganita-sara-sangraha (A.D. 850) or Ganita for short.

It holds the distinction as the oldest known Indian text devoted entirely to mathematics and probably represented all the mathematical knowledge known in India at that time.

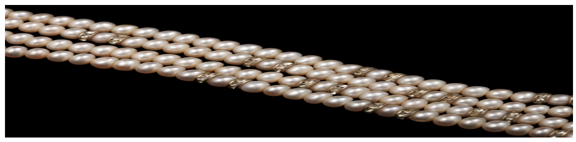
Ganita was written by Mahaviracharya or Mahavira for short, a scholar living in southern India.

Many assertions are made, a few <u>highlights:</u>
The square root of a negative number does not exist.
Properties of Zero
Determining areas and perimeters
Solving linear and quadratic equations

Mar 5-11:04 AM

One of the more interesting problem in the Ganita deals with a young lady and her pearl necklace. The problem is translated as follows:

A young lady has a quarrel with her husband and damages her necklace. One third of the necklace's pearls scatter toward the lady. One-sixth falls on the bed. One-half of what remains (and one-half of what remains thereafter and again one-half of what remains after that, and so on, six times in all) fall everywhere else. A total of 1161 pearls were found to remain unscattered. How many pearls did the woman originally have in total?



Mar 5-11:18 AM

A young lady has a quarrel with her husband and damages her necklace. One third of the necklace's pearls scatter toward the lady. One-sixth falls on the bed.
One-half of what remains (and one-half of what remains thereafter and again one-half of what remains after that, and so on, six times in all) fall everywhere else. A total of 1161 pearls were found to remain unscattered. How many pearls did the woman originally have in total?

<u>TIP:</u>
What is being said here is that 1161 pearls represents the total, halved 7 times.

$\left(\frac{1}{2}\right)^2 \cdot x = 25$

<u>TIP2:</u>
If you knew that 25 represents an original quantity halved twice, how could you set that problem up algebraically?

TIP3:
tip2 = (1/2)^2X = 25

Mar 5-12:29 PM

$$\left(\frac{1}{2}\right)^7 x = 1161$$

$$.0078125x = 1161$$

$$x = 148{,}608$$

Mar 5-1:59 PM

One of the neat formulas with regards to astronomy is known as the **Drake Equation**. It allows us to approximate the number of planets that probably sustain intelligent life. Let's take a look!

http://www.bbc.com/future/story/20120821-how-many-alien-worlds-exist

http://www.youtube.com/watch?feature=player_embedded&v=6AnLznzIjSE#!

Feb 26-11:10 AM

HW OPTIONS:

#1: Research the Drake Equation and write a one page response for Tuesday. Discuss your overall thoughts on it, why do we need math to explore space, your thoughts on astronomy and space exploration and life in the universe outside of our planet.

#2: How important is mathematics to society? If we didn't have math, how do you think society would look? Is it just a bunch of pointless exercises designed to bore and frustrate you, or is there more to the story? (grading on original depth of opinion)

#3: Watch the movie Agora, which is about the life of Hypatia, and write a detailed review of the movie and the mathematics presented in it.

Feb 19-12:01 PM

Drake Equation

Two possibilities exist: either we are alone in the Universe or we are not. Both are equally terrifying. - Arthur C. Clarke

How can we estimate the number of technological civilizations that might exist among the stars? While working as a radio astronomer at the National Radio Astronomy Observatory in Green Bank, West Virginia, Dr. Frank Drake (currently on the Board of the SETI Institute) conceived an approach to bound the terms involved in estimating the number of technological civilizations that may exist in our galaxy. The Drake Equation, as it has become known, was first presented by Drake in 1961 and identifies specific factors thought to play a role in the development of such civilizations. Although there is no unique solution to this equation, it is a generally accepted tool used by the scientific community to examine these factors.
--Frank Drake, 1961

The equation is usually written:

$$N = R^* \bullet f_p \bullet n_e \bullet f_l \bullet f_i \bullet f_c \bullet L$$

Where,

N = The number of civilizations in The Milky Way Galaxy whose electromagnetic emissions are detectable.

R^* =The rate of formation of stars suitable for the development of intelligent life.

f_p = The fraction of those stars with planetary systems.

n_e = The number of planets, per solar system, with an environment suitable for life.

f_l = The fraction of suitable planets on which life actually appears.

f_i = The fraction of life bearing planets on which intelligent life emerges.

f_c = The fraction of civilizations that develop a technology that releases detectable signs of their existence into space.

L = The length of time such civilizations release detectable signals into space.

Within the limits of our existing technology, any practical search for distant intelligent life must necessarily be a search for some manifestation of a distant technology. In each of its last four decadal reviews, the National Research Council has emphasized the relevance and importance of searching for evidence of the electromagnetic signature of distant civilizations. Besides illuminating the factors involved in such a search, the Drake Equation is a simple, effective tool for stimulating intellectual curiosity about the universe around us, for helping us to understand that life as we know it is the end product of a natural, cosmic evolution, and for making us realize how much we are a part of that universe. A key goal of the SETI Institute is to further high quality research that will yield additional information related to any of the factors of this fascinating equation.

Lesson 13 Summary: March 7th

Algebra Concepts Review
Series
Fibonacci

Lesson Handouts

MoMath Thoughts
Geometric Sequence & Series
Solving Rational Equations worksheet (Kuta)

"It has become almost a cliché to remark that nobody boasts of ignorance of literature, but it is socially acceptable to boast ignorance of science and proudly claim incompetence in mathematics."

Richard Dawkins

"Mathematics expresses values that reflect the cosmos, including orderliness, balance, harmony, logic, and abstract beauty."

Deepak Chopra

WARM UP:

1. 2/3x = 9 solve for x

$\left(\frac{3}{2}\right)\left(\frac{2}{3}\right)x = 9\left(\frac{3}{2}\right) = \frac{27}{2}$

2. sqrt(x) = 9 solve for x

$\sqrt{x}^2 = 9^2$

$x = 81$

3. Simplify $(x^2 + 2x^2)^5 - (3x^3)(4x^4) + (11x)^2$

4. Simplify $15x^6/5x^8 + (x^3)^7 - 2x^2 + 2x^3$

$\frac{15x^6}{5x^8}$

5. Imagine you could pick only one math subject you could study in HS, which would you choose and why? Geometry, Algebra, Probability, Statistics, Trigonometry, Calculus, Discrete Math (computer math)

6. Finish the phrase, "It is as easy as....."

$$(3x^2)^5$$

$$3x^{2^5}$$

3) $243x^{10} - 12x^7 + 121x^2$

4) $\frac{3}{x^2} + x^{21} - 2x^2 + 2x^3$

Visual Fun for the Day:


Read over the handout on *your thoughts* on the video we watched Tuesday.

As you do, consider these questions:

1. What does *partial progress* mean in math? If you don't get a problem but spent half an hour trying, is that time wasted?

2. Why do you think math (or a math class) is something that so often upsets people? Do you consider yourself to be one of those people?

3. How do you define mental toughness?

4. If something is NOT obviously entertaining or easy to do, does that make it bad or not worth your time?

5. Do you think there is creativity in mathematics? Why or why not?

Algebraic Fundamentals Review

7) $3 \cdot \frac{v+9}{3} = 8 \cdot 3$

$v+9=24$

$v=15$

9) $\frac{1}{r+3} = \frac{r+10}{r-2}$

$2x=200$

$x=100$

$r-2=(r+3)(r+10)$

$r-2=r^2+13r+30$

$0=r^2+12r+32$

$\frac{0}{r+4}=\frac{(r+4)(r+8)}{r+4}$

$0=r+8$

$r=-8,-4$

So the answer to the last Warm Up question is......

"1 1 2 3!"

Why?

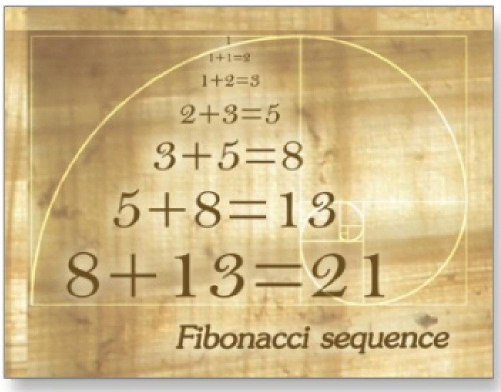
Well, consider the following sequence of integers:

0,1,1,2,3,5,8,13,21,34,55,89

Spend a few minutes and try to determine the pattern. Then try to find a way to explain the pattern.

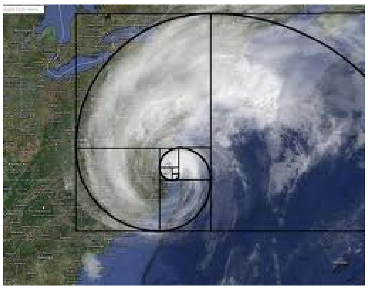
This is called an integer sequence, it is an ordered list of integers. It is one of the most well known sequences, it is called the **Fibonacci Sequence.**

*Now everybody hop on the **one**,*
*the sounds of the **two***
*It's the **third** eye vision, **five** side dimension*
*The **8th** Light, is gonna shine bright tonight*
(Mos Def & Talib Kweli are Black Star)

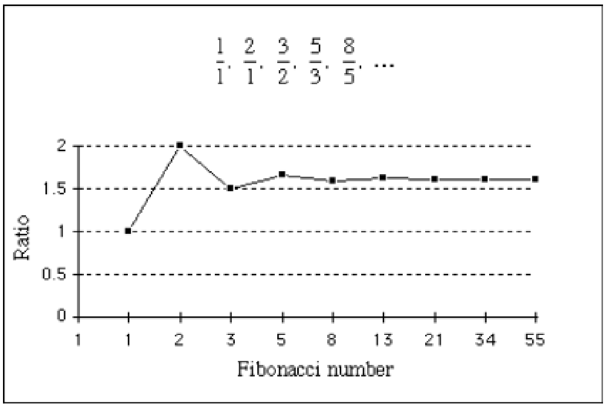


The formula can be found by $F_n = F_{(n-1)} + F_{(n-2)}$. This says that for any term F_n, we find it by adding the previous two terms

Nature is embedded with the Fibonacci sequence, specifically, two consecutive Fibonacci numbers, such as branching in trees, arrangement of leaves on a stem, the fruitlets of a pineapple, the flowering of artichoke, an uncurling fern and the arrangement of a pine cone.



Another way to understand: The series begins with 0 and 1. After that, use the simple rule: Add the last two numbers to get the next.
1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,...
If you take the ratio of two terms of the Fibonacci, as shown below, they approach the value of 1.618, which is known by the Greek symbol Φ, known as phi (fee) also known as the golden ratio.



<u>The Golden Ratio is very prevalent in the following areas:</u>
Architecture
Music
Painting & Art
Nature

What is the first term in the Fibonacci sequence over a million? What numbered term is it?

$F(0)=0$ $F(3)=2$ Φ

$F(1)=1$

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, ...

Midterm Review

Questions? Now is the time to ask!

REMINDER:
Study for Tuesday's Midterm **AND** complete one of the following assignments

#1: Research the Drake Equation and write a one page response for Tuesday. Discuss your overall thoughts on it, why do we need math to explore space, your thoughts on astronomy and space exploration and life in the universe outside of our planet.

#2: How important is mathematics to society? If we didn't have math, how do you think society would look? Is it just a bunch of pointless exercises designed to bore and frustrate you, or is there more to the story? (grading on original depth of opinion)

#3: Watch the movie Agora, which is about the life of Hypatia, and write a detailed review of the movie and the mathematics presented in it.

Highlights of your thoughts on:

Positive: 8
Negative: 2

"I'm surprised it's the first museum of its kind in the U.S. We have museums dedicated to way more frivolous things. It makes sense that we rank very low in math, because as the video said, it is acceptable to be bad at math."

"The most interesting comment is that there is only one traditional pathway for math education."

"Whether we like math or not, it plays a very important part of our lives."

"I think the MoMath museum will definitely have a positive influence on America."

"I think the Math museum is extremely cool."

"MoMath wants to show people that math is this beautiful wonderful thing, and without math we will fall behind in the world of technology and advancements."

"There needs to be more choices in math programs."

"Learners need more patience and should be led to understand the power that math holds."

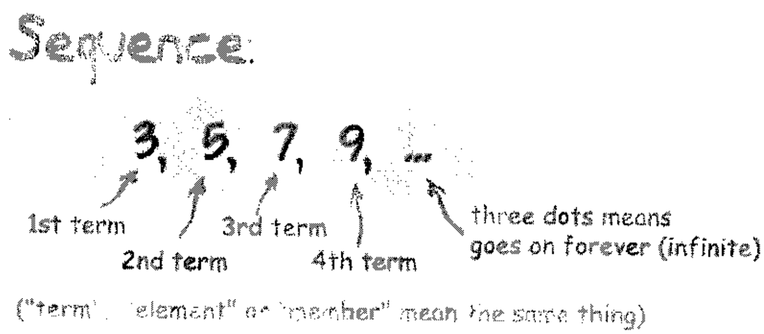
"America is lazy but things like Momath are not going to change how we think about math."

"The brain of someone who enjoys creativity & art will differ from a mathematician."

Geometric Sequences and Sums

Sequence

A Sequence is a set of things (usually numbers) that are in order.



Geometric Sequences

In a **Geometric Sequence** each term is found by **multiplying** the previous term by a **constant.**

Example:

2, 4, 8, 16, 32, 64, 128, 256, ...

This sequence has a factor of 2 between each number.

Each term (except the first term) is found by **multiplying** the previous term by **2.**

In General you could write a Geometric Sequence like this:

$$\{a, ar, ar^2, ar^3, \ldots \}$$

where:

- **a** is the first term, and
- **r** is the factor between the terms (called the **"common ratio"**)

Example: {1,2,4,8,...}

The sequence starts at 1 and doubles each time, so

- **a=1** (the first term)
- **r=2** (the "common ratio" between terms is a doubling)

So we would get:

$$\{a, ar, ar^2, ar^3, \ldots\}$$
$$= \{1, 1\times2, 1\times2^2, 1\times2^3, \ldots\}$$
$$= \{1, 2, 4, 8, \ldots\}$$

But be careful, r should not be 0:

- When **r=0**, you get the sequence {a,0,0,...} which is not geometric

The Rule

You can also calculate **any term** using the Rule:

$$x_n = ar^{(n-1)}$$

(We use "n-1" because ar^0 is for the 1st term)

Example:

10, 30, 90, 270, 810, 2430, ...

This sequence has a factor of 3 between each number.

The values of **a** and **r** are:

- **a = 10** (the first term)
- **r = 3** (the "common ratio")

The Rule for any term is:

$$x_n = 10 \times 3^{(n-1)}$$

So, the **4th** term would be:

$$x_4 = 10\times3^{(4-1)} = 10\times3^3 = 10\times27 = 270$$

And the **10th** term would be:

$$x_{10} = 10\times3^{(10-1)} = 10\times3^9 = 10\times19683 = 196830$$

A Geometric Sequence can also have **smaller and smaller** values:

Example:

Name________________________________

Solving Rational Equations 2

Date________________ Period____

Solve each equation. Remember to check for extraneous solutions.

1) $\frac{k+4}{4} + \frac{k-1}{4} = \frac{k+4}{4k}$

2) $\frac{1}{2m^2} = \frac{1}{m} - \frac{1}{2}$

3) $\frac{n^2-n-6}{n^2} - \frac{2n+12}{n} = \frac{n-6}{2n}$

4) $\frac{3x^2+24x+48}{x^2} + \frac{x-6}{2x^2} = \frac{1}{x^2}$

5) $\frac{k^2+2k-8}{3k^3} = \frac{1}{3k^2} + \frac{1}{k^2}$

6) $\frac{k}{3} - \frac{1}{3k} = \frac{1}{k}$

7) $\frac{x-4}{6x} + \frac{x^2-3x-10}{6x} = \frac{x-1}{6}$

8) $\frac{1}{x^2} = \frac{x-1}{x} + \frac{1}{x}$

9) $\frac{1}{r+3} = \frac{r+4}{r-2} + \frac{6}{r-2}$

10) $\frac{a^2 - 4a - 12}{a^2 - 10a + 25} = \frac{6}{a-5} + \frac{a-3}{a-5}$

11) $\frac{1}{n+3} + \frac{n^2 + 6n + 5}{n+3} = n - 3$

12) $\frac{1}{2} = \frac{x^2 - 7x + 10}{4x} - \frac{1}{2x}$

13) $\frac{1}{k} = 5 + \frac{1}{k^2 + k}$

14) $\frac{1}{p^2 - 4p} + 1 = \frac{p-6}{p}$

15) $\frac{5}{n} - \frac{6}{n^3 - 2n^2} = \frac{n^2 + 5n - 6}{n^3 - 2n^2}$

16) $\frac{x+2}{x} = \frac{x-1}{x} - \frac{4x+2}{x^2 - 3x}$

 Name________________________

Solving Rational Equations 2

Date________________ Period____

Solve each equation. Remember to check for extraneous solutions.

1) $\frac{k+4}{4} + \frac{k-1}{4} = \frac{k+4}{4k}$

$\{-2, 1\}$

2) $\frac{1}{2m^2} = \frac{1}{m} - \frac{1}{2}$

$\{1\}$

3) $\frac{n^2 - n - 6}{n^2} - \frac{2n + 12}{n} = \frac{n - 6}{2n}$

$\left\{-\frac{2}{3}, -6\right\}$

4) $\frac{3x^2 + 24x + 48}{x^2} + \frac{x - 6}{2x^2} = \frac{1}{x^2}$

$\left\{-\frac{8}{3}, -\frac{11}{2}\right\}$

5) $\frac{k^2 + 2k - 8}{3k^3} = \frac{1}{3k^2} + \frac{1}{k^2}$

$\{-2, 4\}$

6) $\frac{k}{3} - \frac{1}{3k} = \frac{1}{k}$

$\{-2, 2\}$

7) $\frac{x-4}{6x} + \frac{x^2 - 3x - 10}{6x} = \frac{x-1}{6}$

$\{-14\}$

8) $\frac{1}{x^2} = \frac{x-1}{x} + \frac{1}{x}$

$\{1, -1\}$

9) $\frac{1}{r+3} = \frac{r+4}{r-2} + \frac{6}{r-2}$

$\{-8, -4\}$

10) $\frac{a^2 - 4a - 12}{a^2 - 10a + 25} = \frac{6}{a-5} + \frac{a-3}{a-5}$

$\left\{\frac{3}{2}\right\}$

11) $\frac{1}{n+3} + \frac{n^2 + 6n + 5}{n+3} = n - 3$

$\left\{-\frac{5}{2}\right\}$

12) $\frac{1}{2} = \frac{x^2 - 7x + 10}{4x} - \frac{1}{2x}$

$\{1, 8\}$

13) $\frac{1}{k} = 5 + \frac{1}{k^2 + k}$

$\left\{-\frac{4}{5}\right\}$

14) $\frac{1}{p^2 - 4p} + 1 = \frac{p-6}{p}$

$\left\{\frac{23}{6}\right\}$

15) $\frac{5}{n} - \frac{6}{n^3 - 2n^2} = \frac{n^2 + 5n - 6}{n^3 - 2n^2}$

$\left\{\frac{15}{4}\right\}$

16) $\frac{x+2}{x} = \frac{x-1}{x} - \frac{4x+2}{x^2 - 3x}$

$\{1\}$

Lesson 14 Summary: March 12
MIDTERM

Lesson 15 Summary: March 14th
Pi Day
Rational Equations
Geometric Progressions

Lesson Handouts
Wheat on a Chessboard (Cliff Pickover, The Math Book)

-- SPRINGBREAK --

"God used beautiful mathematics in creating the world."
Paul Dirac

"If a man's wit be wandering, let him study the mathematics."
Francis Bacon

WARM UP:

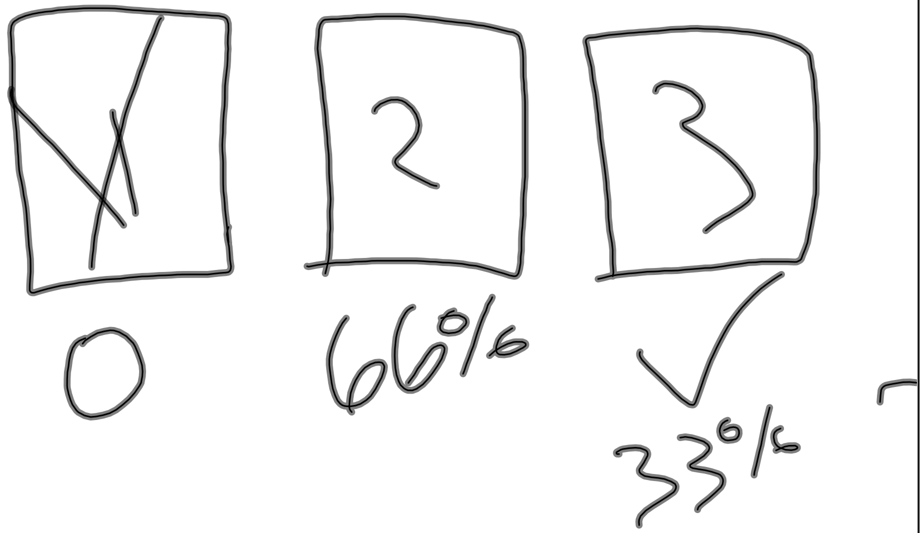
1. How did you find the midterm?

2. Do you have a favorite number? If yes, what is it and why?

3. Have you heard about the Monty Hall problem? If so, what is it?

4. List all the ways that you will consciously and unconsciously use mathematics over the spring break.

5. Why has there never been a scientist in a high political office? Why do you think that is? Do you think that a mathematician/scientist would be an effective leader? Why or why not?



Today is March 14, also known as...

Pi Day

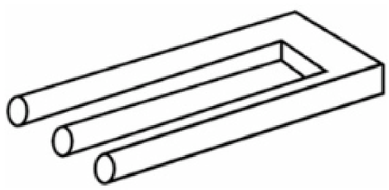
What is Pi?

To have a little fun, I will show you 314 digits of Pi for a few minutes. You will then try to write down as many digits as you can remember. Whoever is the most accurate will win a small prize.

3.141592653589793238462643383279
5028841971693993751058209749445
9230781640628620899862803482534
2117067982148086513282306647093
8446095505822317253594081284811
1745028410270193852110555964462
2948954930381964428810975665933
4461284756482337867831652712019
0914564856692346034861045432664
8213393607260249141273724587006
6063

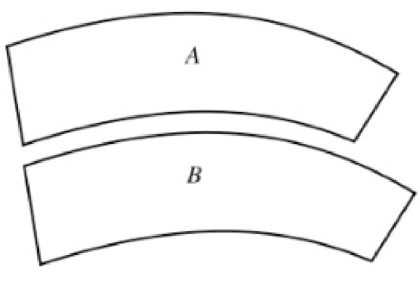
Visual Fun for the Day:

Blivet



A blivet, also known as a poiuyt, is an undecipherable figure, an optical illusion and an impossible object. It appears to have three cylindrical prongs at one end which then mysteriously transform into two rectangular prongs at the other end.

The Jastrow illusion



The Jastrow illusion is an optical illusion discovered by the American psychologist Joseph Jastrow in 1889. In this illustration, the two figures are identical, although the lower one appears to be larger.

A few of your Thoughts (March 7th)

"Mental toughness is the ability to keep moving forward and trying again and again regardless of the circumstances."

Is there creativity in mathematics, heavens yes! To find "elegant" proofs, to see the way to a solution, to recognize patterns, these are the height of creativity!"

"Partial progress is a very important aspect of the problem solving process."

MIDTERM RESULTS

There was a perfect midterm.

The average was a 70%.

Let's take a moment, go over the midterm answers and individually we can review your grades to date.

So, the algebra worksheet from last week, which we did not complete together, apparently has some challenging examples, even the tutoring center could not solve! Let's try a few together.

Specifically, lets work on the "solving rational equations" problems 5,7,10, 13,16.

Here is a tip to doing these problems. You want to eliminate the fractions, so find the common denominator and rewrite all the numerators. Consider that at this point, the denominators are the same. So do they really matter? Not really (other than for saying what values x can't be). So at this point, the two sides of the equation will be equal as long as the numerators are equal. That is, all I really need to do now is solve the numerators.

$$\frac{k^2+2k-8}{3k^3} = \frac{1}{3k^2} + \frac{1}{k^2}$$

$$k^2+2k-8 = k + 3k$$

$$k^2-2k-8=0$$

$$(k+2)(k-4)=0$$

$$k=-2,4$$

$$\frac{1}{r+3} = \frac{r+4}{r-2} + \frac{6}{r-2}$$

$$\frac{1}{r+3} = \frac{r+10}{r-2} \qquad \frac{1}{4} + \frac{2}{4} = \boxed{\frac{3}{4}}$$

$$r-2 = (r+3)(r+10)$$

$$r-2 = r^2+13r+30$$

$$r^2+12r+32=0$$

$$(r+4)(r+8)=0$$

$$r=(-4,-8)$$

In mathematics, a **geometric progression**, also known as a geometric sequence, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the *common ratio*. For example, the sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3. Similarly 10, 5, 2.5, 1.25, ... is a geometric sequence with common ratio 1/2.

The first term of a geometric sequence is 192 and the fifth term is 0.75. What is the common ratio?

HOMEWORK:

Remember, original thoughts only! Over the Spring break, please do one of the following:

1. Find a movie that centers around mathematics, watch it and write a review. (Not Agora!)

2. Write about Pi; its importance, its history, and its development through history.

3. Research and write about the competitor to Pi, known as Tau. (tauday.com has the manifesto)

$$X_N = Ar^{(N-1)}$$

$$X_5 = 1 \cdot 2^4$$

$$X_5 = 16$$

What is the ninth term of the geometric sequence 81, 27, 9, 3, ... ?

$$\frac{1}{3} = 3^{-1}$$

What is the eleventh term of the geometric sequence 3, 6, 12, 24, ... ?

This sequence has a factor of 2 between each pair of numbers, so the values of a and r are:
- a = 3 (the first term)
- r = 2 (the "common ratio")

The Rule for any term is:
$x_n = ar^{n-1}$

Therefore $x_{11} = 3 \times 2^{10} = 3 \times 1{,}024 = 3{,}072$

The fourth term of a geometric sequence is 27 and the seventh term is 1. What is the first term?

$X_N = Ar^{(N-1)}$

$27 = ar^3$

$1 = ar^6$

$\frac{1}{27} = \frac{ar^6}{ar^3}$

$\frac{1}{27} = r^3$ $\quad \frac{1}{3^3} =$ $\quad r = \frac{1}{3}$

Wheat on a Chessboard

Abu-l 'Abbas Ahmad ibn Khallikan (1211–1282), Dante Alighieri (1265–1321)

The problem of Sissa's chessboard is notable in the history of mathematics because it has been used for centuries to demonstrate the nature of geometric growth or geometric progressions, and it is one of the earliest mentions of chess in puzzles. The Arabic scholar Ibn Khallikan in 1256 appears to be the first author to discuss the story of Grand Vizier Sissa ben Dahir, who, according to legend, was asked by the Indian King Shirham what reward he wanted for inventing the game of chess.

Sissa addressed the king: "Majesty, I would be happy if you were to give me a grain of wheat to place on the first square of the chessboard, and two grains of wheat to place on the second square, and four grains of wheat to place on the third, and eight grains of wheat to place on the fourth, and so on for the sixty-four squares."

"And is that all you wish, Sissa, you fool?" the astonished King shouted.

The king did not realize how many grains Sissa would be awarded! One way to determine the solution is to compute the sum of the first 64 terms of a geometrical progression, $1 + 2 + 2^2 + \ldots + 2^{63} = 2^{64} - 1$, which is a walloping 18,446,744,073,709,551,615 grains of wheat.

It is possible that some version of this story was known to Dante, because he referred to a similar concept in the *Paradiso* to describe the abundance of Heaven's lights: "They were so many that their number piles up faster than the chessboard doubling." Jan Gullberg writes, "With about 100 grains to a cubic centimeter, the total volume of [Sissa's] wheat would be nearly…two hundred cubic kilometers, to be loaded on two thousand million railway wagons, which would make up a train reaching a thousand times around the Earth."

SEE ALSO Harmonic Series Diverges (c. 1350), Rope around the Earth Puzzle (1702), and Rubik's Cube (1974).

The famous problem of Sissa's chessboard demonstrates the nature of geometric progressions. In the smaller version depicted here, how many pieces of candy will the hungry beetle get if the progression 1 + 2 + 4 + 8 + 16 . . . continues?

Lesson 16 Summary: March 26th
Extended Warm Up (Probability)
Series

Lesson Handouts
Harmonic Series Diverge (Cliff Pickover, The Math Book)
Infinite Geometric Series worksheet (Kuta)
Finite Geometric Series worksheet (Kuta)

"Mathematics is not about numbers, equations, computations, or algorithms: *it is about understanding*."

---William Paul Thurston

WARM UP (extended):

1. Solve for x: x/(x-2) + 1/(x-4) = 2/(x^2 - 6x + 8)

2. If a soda and a slice of pizza cost $1.50 together and the slice costs $1.00 more than the soda how much does the soda cost?

3. In a standard deck of cards, what is the least amount of cards you must take to be *guaranteed* at least one four-of-a-kind?

4. There are 30 children in a class and they all have at least one cat or dog.14 children have a cat, 19 children have a dog. What is the probability that a child chosen at random from the class has both a cat and a dog?

5. In a group of 25 boys, 20 play ice hockey and 17 play baseball. They all play at least one of the games. What is the probability that a boy chosen at random from the class plays ice hockey but not baseball?

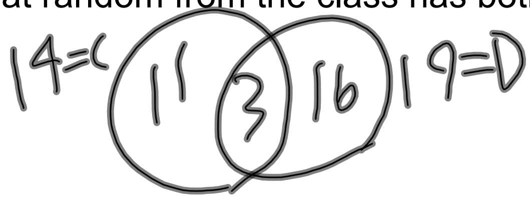
WARM UP (extended):

3. In a standard deck of cards, what is the least amount of cards you must take to be *guaranteed* at least one four-of-a-kind?

13 A
13 Q

4. There are 30 children in a class and they all have at least one cat or dog.14 children have a cat, 19 children have a dog. What is the probability that a child chosen at random from the class has both a cat and a dog?

$\frac{3}{30}$



5. In a group of 25 boys, 20 play ice hockey and 17 play baseball. They all play at least one of the games. What is the probability that a boy chosen at random from the class plays ice hockey but not baseball?

WARM UP (extended):

1. Solve for x: x/(x-2) + 1/(x-4) = 2/(x^2 - 6x + 8)

$(x-4)(x-2)$

$x(x-4) + x-2 = 2$

$x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1)$

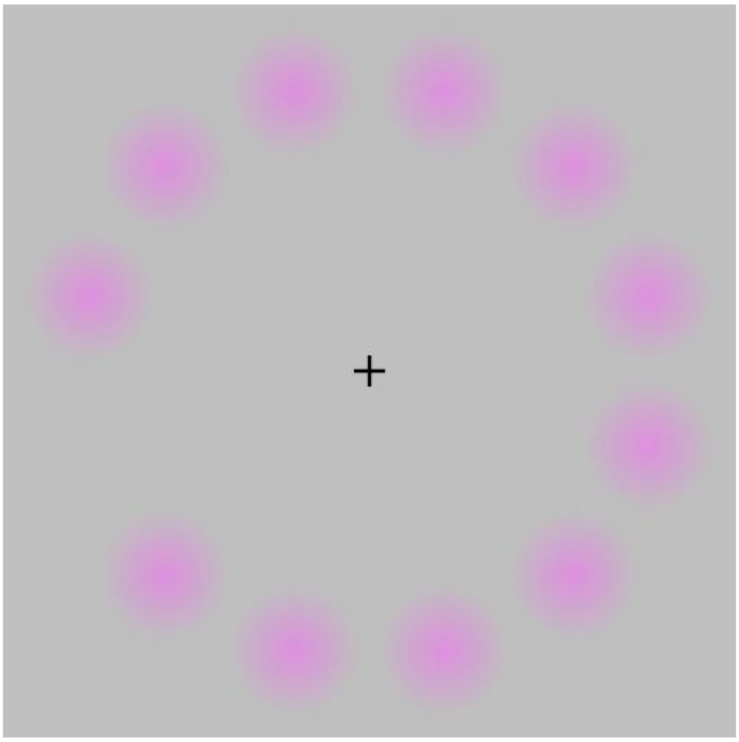
$x = -1$

2. If a soda and a slice of pizza cost $1.50 together and the slice costs $1.00 more than the soda how much does the soda cost?

$x + x + 1 = 1.5$

Visual Fun for the Day:

Stare at the crosshair long enough and the pink dots seem to disappear.



A few of your Thoughts (March 7th)

"Mental toughness is the ability to keep moving forward and trying again and again regardless of the circumstances."

Is there creativity in mathematics, heavens yes! To find "elegant" proofs, to see the way to a solution, to recognize patterns, these are the height of creativity!"

"Partial progress is a very important aspect of the problem solving process."

In the past few lessons, we have considered the Fibonacci sequence, (1,1,2,3,5,8,13,21...) and geometric sequences (ie 2,6,18,54,162...).

One thing mathematicians like to do when studying sequences is to consider the **series** of a sequence. A **series** is just the sum of the terms in the sequence. Series typically are classified as either **"divergent"** or **"convergent"**.

In a general sense we can say two paths are convergent if they come together and two paths are divergent if they get further and further apart. So what do they mean with series?

Convergent series are those that are finite, that arrive at a fixed value and divergent series are those that grow without bound, aka head towards infinity.

When working with series, since we are talking about sums, we have a special symbol that we use to indicate to the reader that we are doing a summation of a sequence.

$$\Sigma$$

More specifically, it will look like this:

Sum ends with this number

$$\sum_{k=1}^{n=7} a_k \quad 4^{k-1} \rightarrow 4^{1-1} = 1$$

Sum starts with this number

So for problem 5, the terms of the series will be: $4^0 + 4^1 + 4^2 + 4^3 + 4^4 + 4^5 + 4^6 = ?$ 5461

Infinite Geometric Series will converge when the absolute value of R < 1, else the series will diverge!

Divergence implies no sum, aka ∞.

Given convergence, we can use the following formula to evaluate the summation:

$a_1/(1-r)$

$$\frac{3}{1-(-\frac{1}{5})} = \frac{3}{1.2} = 2.5$$

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$$

$$r = \frac{1}{2}$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \quad \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$3r = \frac{-9}{4}$$

$$r = \frac{-9}{4} \div 3$$

$$= -\frac{9}{4} \cdot \frac{1}{3}$$

$$r = -\frac{3}{4}$$

$$S = \frac{A_1}{1-r}$$

$$1.25 = \frac{1}{1-r}$$

$$1.25(1-r) = 1$$

$$1 - r = \frac{1}{1.25}$$

$$1 - \frac{1}{1.25} = r$$

$$r = 1 - .8$$

$$= .2$$

c. 1350

Harmonic Series Diverges

Nicole Oresme (1323–1382), **Pietro Mengoli** (1626–1686), **Johann Bernoulli** (1667–1748), **Jacob Bernoulli** (1654–1705)

If God were infinity, then *divergent series* would be His angels flying higher and higher to reach Him. Given an eternity, all such angels approach their Creator. For example, consider the following infinite series: 1 + 2 + 3 + 4 . . . . If we add one term of the series each year, in four years the sum will be 10. Eventually, after an infinite number of years, the sum reaches infinity. Mathematicians call such series divergent because they explode to infinity, given an infinite number of terms. For this entry, we are interested in a series that diverges much more slowly. We're interested in a more magical series, an angel, perhaps, with weaker wings.

Consider the harmonic series, the first famous example of a divergent series whose terms approach zero: 1 + 1/2 + 1/3 + 1/4 + ... Of course, this series explodes more slowly than does our previous example, but it still grows to infinity. In fact, it grows so incredibly slowly that if we added a term a year, in 10^{43} years we'd have a sum less than 100. William Dunham writes, "Seasoned mathematicians tend to forget how surprising this phenomenon appears to the uninitiated student—that, by adding ever more negligible terms, we nonetheless reach a sum greater than any preassigned quantity."

Nicole Oresme, the famous French philosopher of the Middle Ages, was the first to prove the divergence of the harmonic series (c. 1350). His results were lost for several centuries, and the result was proved again by Italian mathematician Pietro Mengoli in 1647 and by Swiss mathematician Johann Bernoulli in 1687. His brother Jacob Bernoulli published a proof in his 1689 work *Tractatus de Seriebus Infinitis* (*Treatise on Infinite Series*), which he closes with: "So the soul of immensity dwells in minutia. And in narrowest limits no limits inhere. What joy to discern the minute in infinity! The vast to perceive in the small, what divinity!"

SEE ALSO Zeno's Paradoxes (c. 445 B.C.), Wheat on a Chessboard (1256), Discovery of Series Formula for π (c. 1500), Brun's Constant (1919), and Polygon Circumscribing (c. 1940).

Depiction of Nicole Oresme from his Tractatus de origine, natura, jure et mutationibus monetarum (On the Origin, Nature, Juridical Status, and Variations of Coinage), *which was published around the year 13*

Name________________________

Infinite Geometric Series

Date________________ Period____

Determine if each geometric series converges or diverges.

1) $a_1 = -3,\ r = 4$

2) $a_1 = 4,\ r = -\frac{3}{4}$

3) $a_1 = 5.5,\ r = 0.5$

4) $a_1 = -1,\ r = 3$

5) $81 + 27 + 9 + 3...,$

6) $7.1 + 17.75 + 44.375 + 110.9375...,$

7) $-3 + \frac{12}{5} - \frac{48}{25} + \frac{192}{125}...,$

8) $\frac{128}{3125} - \frac{64}{625} + \frac{32}{125} - \frac{16}{25}...,$

9) $\sum_{k=1}^{\infty} -4^{k-1}$

10) $\sum_{k=1}^{\infty} \frac{16}{9}\left(\frac{3}{2}\right)^{k-1}$

11) $\sum_{i=1}^{\infty} 4.2 \cdot 0.2^{i-1}$

12) $\sum_{k=1}^{\infty} \frac{7}{6}\left(-\frac{1}{4}\right)^{k-1}$

Evaluate each infinite geometric series described.

13) $a_1 = 3,\ r = -\frac{1}{5}$

14) $a_1 = 1,\ r = -4$

15) $a_1 = 1,\ r = -3$

16) $a_1 = 1,\ r = \frac{1}{2}$

17) $1 + 0.5 + 0.25 + 0.125...,$

18) $3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64}...,$

19) $81 - 27 + 9 - 3...,$

20) $1 - 0.6 + 0.36 - 0.216...,$

21) $\sum_{k=1}^{\infty} 5 \cdot \left(-\frac{1}{5}\right)^{k-1}$

22) $\sum_{n=1}^{\infty} -6 \cdot \left(-\frac{1}{2}\right)^{n-1}$

23) $\sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i-1}$

24) $\sum_{k=1}^{\infty} 4^{k-1}$

Determine the common ratio of the infinite geometric series.

25) $a_1 = 1,\ S = 1.25$

26) $a_1 = 96,\ S = 64$

27) $a_1 = -4,\ S = -\frac{16}{5}$

28) $a_1 = 1,\ S = 2.5$

Name________________________

Infinite Geometric Series

Date________________ Period____

Determine if each geometric series converges or diverges.

1) $a_1 = -3,\ r = 4$

Diverges

2) $a_1 = 4,\ r = -\frac{3}{4}$

Converges

3) $a_1 = 5.5,\ r = 0.5$

Converges

4) $a_1 = -1,\ r = 3$

Diverges

5) $81 + 27 + 9 + 3...,$

Converges

6) $7.1 + 17.75 + 44.375 + 110.9375...,$

Diverges

7) $-3 + \frac{12}{5} - \frac{48}{25} + \frac{192}{125}...,$

Converges

8) $\frac{128}{3125} - \frac{64}{625} + \frac{32}{125} - \frac{16}{25}...,$

Diverges

9) $\sum_{k=1}^{\infty} -4^{k-1}$

Diverges

10) $\sum_{k=1}^{\infty} \frac{16}{9}\left(\frac{3}{2}\right)^{k-1}$

Diverges

11) $\sum_{i=1}^{\infty} 4.2 \cdot 0.2^{i-1}$

Converges

12) $\sum_{k=1}^{\infty} \frac{7}{6}\left(-\frac{1}{4}\right)^{k-1}$

Converges

Evaluate each infinite geometric series described.

13) $a_1 = 3,\ r = -\frac{1}{5}$

$\frac{5}{2}$

14) $a_1 = 1,\ r = -4$

No sum

15) $a_1 = 1,\ r = -3$

No sum

16) $a_1 = 1,\ r = \frac{1}{2}$

2

17) $1 + 0.5 + 0.25 + 0.125...,$

2

18) $3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64}...,$

$\frac{12}{7}$

19) $81 - 27 + 9 - 3...,$

$\frac{243}{4}$

20) $1 - 0.6 + 0.36 - 0.216...,$

0.625

21) $\sum_{k=1}^{\infty} 5 \cdot \left(-\frac{1}{5}\right)^{k-1}$

$\frac{25}{6}$

22) $\sum_{n=1}^{\infty} -6 \cdot \left(-\frac{1}{2}\right)^{n-1}$

-4

23) $\sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i-1}$

$\frac{3}{2}$

24) $\sum_{k=1}^{\infty} 4^{k-1}$

No sum

Determine the common ratio of the infinite geometric series.

25) $a_1 = 1,\ S = 1.25$

0.2

26) $a_1 = 96,\ S = 64$

$-\frac{1}{2}$

27) $a_1 = -4,\ S = -\frac{16}{5}$

$-\frac{1}{4}$

28) $a_1 = 1,\ S = 2.5$

0.6

 Name________________________

Finite Geometric Series

Date________________ Period____

Evaluate the related series of each sequence.

1) 2, 12, 72, 432

2) −1, 5, −25, 125

3) −2, 6, −18, 54, −162

4) −2, −12, −72, −432, −2592

Evaluate each geometric series described.

5) $\sum_{k=1}^{7} 4^{k-1}$

6) $\sum_{i=1}^{8} (-6)^{i-1}$

7) $\sum_{i=1}^{9} 2^{i-1}$

8) $\sum_{m=1}^{9} -2^{m-1}$

9) $\sum_{n=1}^{8} 2 \cdot (-2)^{n-1}$

10) $\sum_{n=1}^{9} 4 \cdot 3^{n-1}$

11) $\sum_{n=1}^{10} 4 \cdot (-3)^{n-1}$

12) $\sum_{n=1}^{9} (-2)^{n-1}$

13) $1 + 2 + 4 + 8...,\ n = 6$

14) $2 - 10 + 50 - 250...,\ n = 8$

15) $1 - 4 + 16 - 64...,\ n = 9$

16) $-2 - 6 - 18 - 54...,\ n = 9$

17) $1 - 5 + 25 - 125...,\ n = 7$

18) $-3 - 6 - 12 - 24...,\ n = 9$

19) $a_1 = 4,\ a_n = 1024,\ r = -2$

20) $a_1 = 4,\ a_n = 8748,\ r = 3$

Determine the number of terms *n* in each geometric series.

21) $a_1 = -2,\ r = 5,\ S_n = -62$

22) $a_1 = 3,\ r = -3,\ S_n = -60$

23) $a_1 = -3,\ r = 4,\ S_n = -4095$

24) $a_1 = -3,\ r = -2,\ S_n = 63$

25) $-4 + 16 - 64 + 256...,\ S_n = 52428$

26) $\sum_{m=1}^{n} -2 \cdot 4^{m-1} = -42$

Name________________________________

Finite Geometric Series

Date__________________ Period____

Evaluate the related series of each sequence.

1) $2, \; 12, \; 72, \; 432$

518

2) $-1, \; 5, \; -25, \; 125$

104

3) $-2, \; 6, \; -18, \; 54, \; -162$

-122

4) $-2, \; -12, \; -72, \; -432, \; -2592$

-3110

Evaluate each geometric series described.

5) $\sum_{k=1}^{7} 4^{k-1}$

5461

6) $\sum_{i=1}^{8} (-6)^{i-1}$

-239945

7) $\sum_{i=1}^{9} 2^{i-1}$

511

8) $\sum_{m=1}^{9} -2^{m-1}$

-511

9) $\sum_{n=1}^{8} 2 \cdot (-2)^{n-1}$

-170

10) $\sum_{n=1}^{9} 4 \cdot 3^{n-1}$

39364

11) $\sum_{n=1}^{10} 4 \cdot (-3)^{n-1}$

-59048

12) $\sum_{n=1}^{9} (-2)^{n-1}$

171

13) $1 + 2 + 4 + 8...,\ n = 6$

63

14) $2 - 10 + 50 - 250...,\ n = 8$

-130208

15) $1 - 4 + 16 - 64...,\ n = 9$

52429

16) $-2 - 6 - 18 - 54...,\ n = 9$

-19682

17) $1 - 5 + 25 - 125...,\ n = 7$

13021

18) $-3 - 6 - 12 - 24...,\ n = 9$

-1533

19) $a_1 = 4,\ a_n = 1024,\ r = -2$

684

20) $a_1 = 4,\ a_n = 8748,\ r = 3$

13120

Determine the number of terms *n* in each geometric series.

21) $a_1 = -2,\ r = 5,\ S_n = -62$

3

22) $a_1 = 3,\ r = -3,\ S_n = -60$

4

23) $a_1 = -3,\ r = 4,\ S_n = -4095$

6

24) $a_1 = -3,\ r = -2,\ S_n = 63$

6

25) $-4 + 16 - 64 + 256...,\ S_n = 52428$

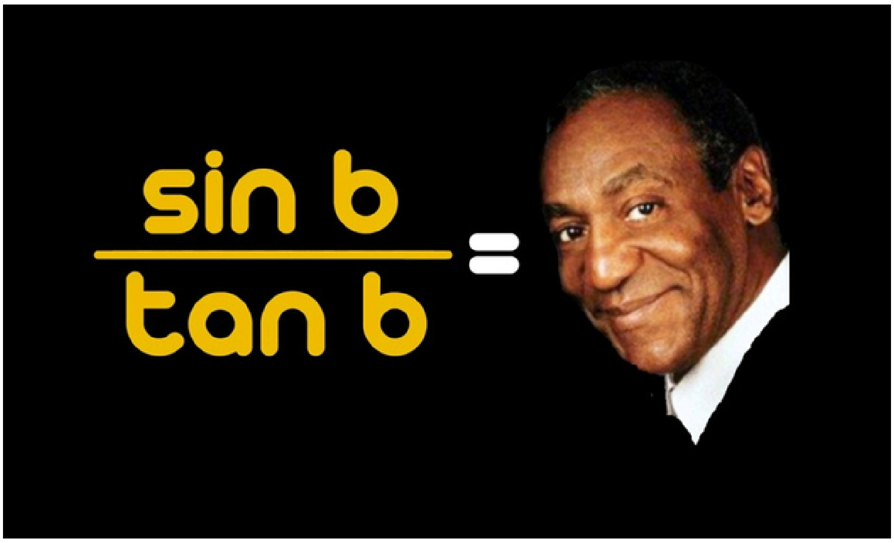
8

26) $\sum_{m=1}^{n} -2 \cdot 4^{m-1} = -42$

3

Lesson 17 Summary: March 28th
Venn Diagrams
History of Trigonometry
Law of Cosines

Lesson Handouts
Trigonometric Ratios worksheet (Kuta)
The Law of Cosines worksheet (Kuta)



WARM UP:

13 H, 13D, 13C, 13S

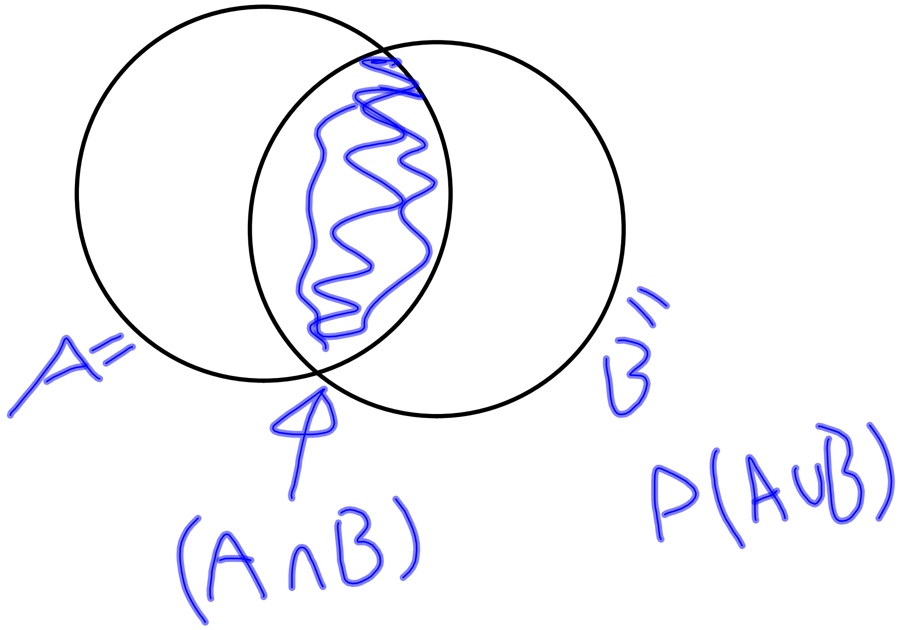
1. A card is chosen at random from a pack of 52 playing cards.What is the probability of a King or a Heart?

2. In a class of 29 children, 15 like history and 21 like math. They all like at least one of the two subjects.What is the probability that a child chosen at random from the class likes math but not history?

3. HARD: In a class of 35 children, 22 like bananas, 18 like cherries and 13 like strawberries. 7 of them like bananas and cherries. 8 of them like bananas and strawberries. 5 of them like cherries and strawberries. They all like at least one of the fruits. What is the probability that a child chosen at random from the class likes cherries only?

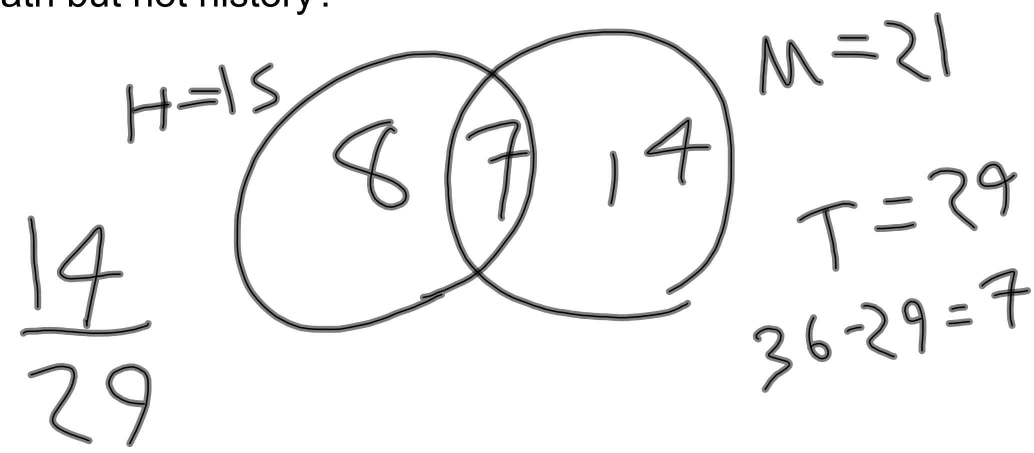
HINT1: DRAW, DRAW, DRAW

HINT2: $P(B \cup C \cup S) = 35 = P(B) + P(C) + P(S) - P(B \cap C) - P(B \cap S) - P(C \cap S) + P(B \cap C \cap S)$



WARM UP:

2. In a class of 29 children, 15 like history and 21 like math. They all like at least one of the two subjects.What is the probability that a child chosen at random from the class likes math but not history?



WARM UP:

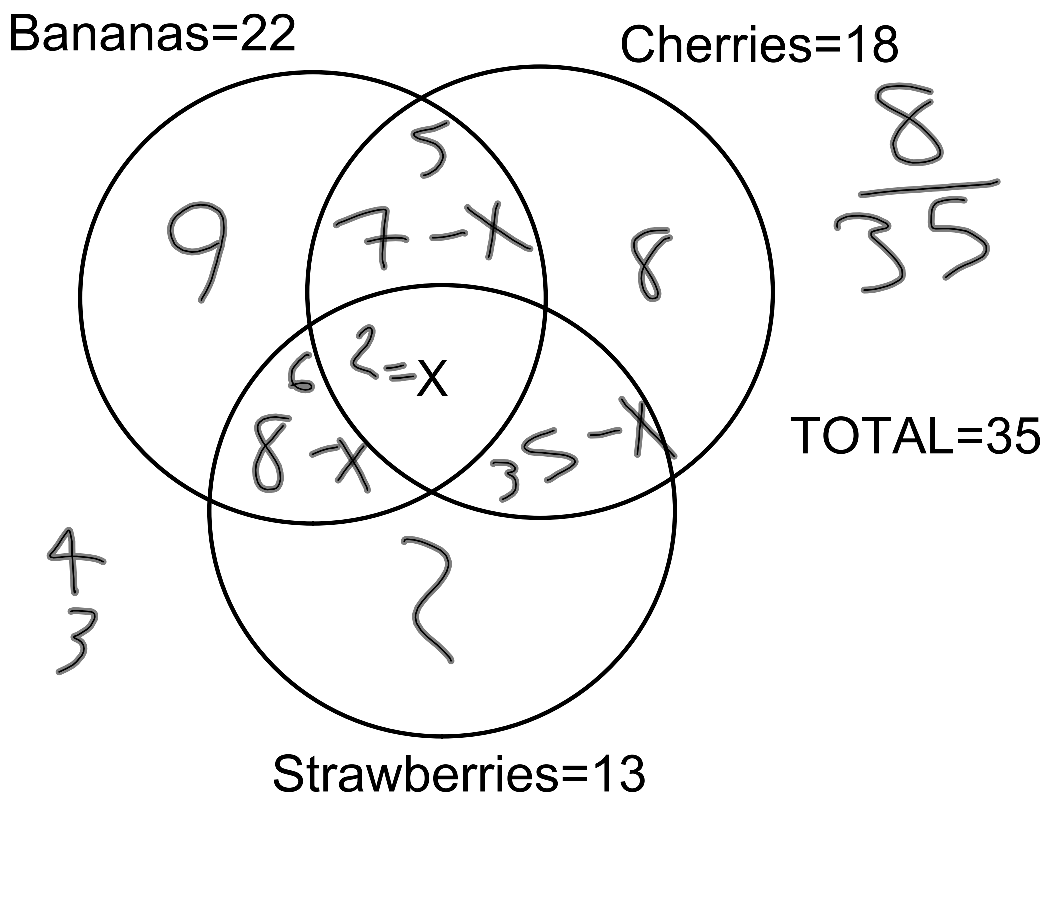
3. HARD: In a class of 35 children, 22 like bananas, 18 like cherries and 13 like strawberries. 7 of them like bananas and cherries. 8 of them like bananas and strawberries. 5 of them like cherries and strawberries. They all like at least one of the fruits. What is the probability that a child chosen at random from the class likes cherries only?

HINT1: DRAW, DRAW, DRAW

HINT2: $P(B \cup C \cup S) = 35 = P(B) + P(C) + P(S) - P(B \cap C) - P(B \cap S) - P(C \cap S) + P(B \cap C \cap S)$

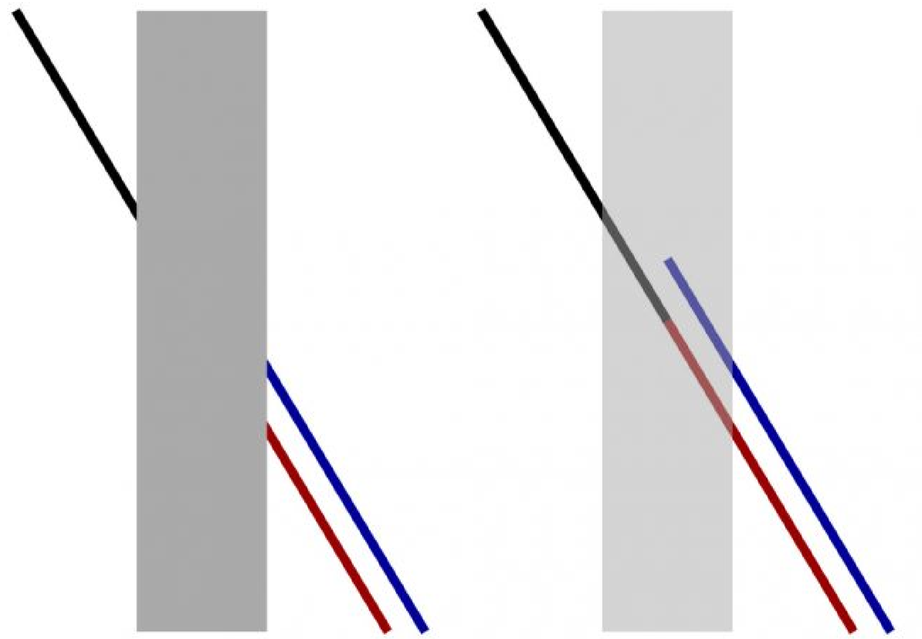
$35 = 53 - (7 + 8 + 5) + X$

$X = 2$



Visual Fun for the Day:

The Poggendorff illusion: Does the black line connect to the blue or the red line?



Let's confirm the answers to the homework questions on Series.

$$2.5 = \frac{1}{1-r} \quad (\times 1-r)$$

$$2.5(1-r) = 1$$

$$1-r = \frac{1}{2.5}$$

$$r = 1 - \frac{1}{2.5}$$

$$=$$

One of the conceptually trickier areas of mathematics is **Trigonometry**. *Trigonometry is a branch of mathematics that studies triangles and the relationships between the lengths of their sides and the angles between those sides.*

Fields that use trigonometry or trigonometric functions include astronomy (especially for locating apparent positions of celestial objects), navigation (on the oceans, in aircraft, and in space), music theory, acoustics, optics, analysis of financial markets, electronics, probability theory, statistics, biology, medical imaging (CAT scans and ultrasound), pharmacy, chemistry, number theory (and hence cryptology), seismology, meteorology, oceanography, many physical sciences, land surveying and geodesy, architecture, phonetics, economics, electrical engineering, mechanical engineering, civil engineering, computer graphics, cartography, crystallography and game development.

HISTORY OF TRIGONOMETRY

- Sumerian astronomers introduced angle measure, using a division of circles into 360 degrees.

- Classical Greek mathematicians (such as Euclid and Archimedes) studied the properties of chords and inscribed angles in circles.

- Ptolemy expanded upon Hipparchus' Chords in a Circle in his Almagest.

- The modern sine function was first defined in the Surya Siddhanta, and its properties were further documented by the 5th century Indian mathematician and astronomer Aryabhata.

- By the 10th century, Islamic mathematicians were using all six trigonometric functions, had tabulated their values, and were applying them to problems in spherical geometry.

- One of the earliest works on trigonometry by a European mathematician is De Triangulis by the 15th century German mathematician Regiomontanus.

- Driven by the demands of navigation and the growing need for accurate maps of large areas, trigonometry grew into a major branch of mathematics.

Let's start with the most basic application of Trigonometry.

This is understanding the basic definitions of the 3 main trigonometric functions in the context of a right triangle.

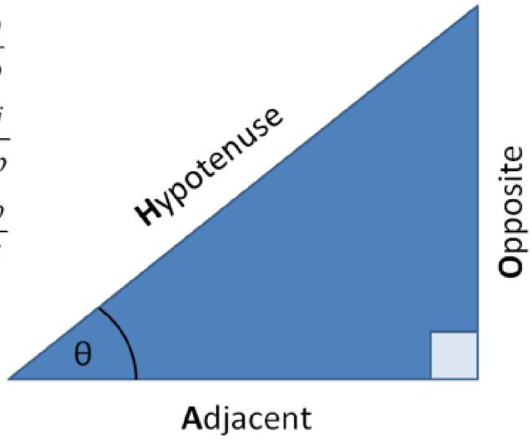
They are:

SOHCAHTOA

$$\sin(\theta) = \frac{Opp}{Hyp}$$

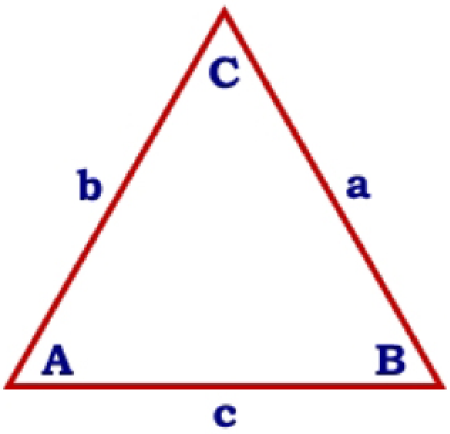
$$\cos(\theta) = \frac{Adj}{Hyp}$$

$$\tan(\theta) = \frac{Opp}{Adj}$$



One of the more common trigonometric laws is called the **Law of Cosines.** It allows us to calculate, for any triangle, the length of a side of a triangle when we know two sides and the angle opposite the side we want to know.

The Pythagorean Theorem is actually a special case of the Law of Cosines, when C is 90.



$$c^2 = a^2 + b^2 - 2\,ab\,\text{Cos}\,C$$

Euclid's elements contained the basic idea of the **Law of Cosines**, but it wasn't until the fifteenth century when the Persian astronomer al-Kashi formalized the theorem. This can be found in his work, *"The Key to Arithmetic"*, completed in 1427. A French mathematician, Francois Viete also discovered the law independently of al-Kashi. Viete once cracked a complex cipher of the King of Spain for the King of France. When the King of Spain learned that the French knew of his military plans, he complained to the Pope that black magic was being used against Spain.

Let's now use the Law of Cosines to make some computations.

Be sure to locate the COS button on your given calculator.

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

Plug in what you know. Remember, we want to solve for C!

$c^2 = 29^2 + 13^2 - 2(29)(13)*\text{Cos}(41 \text{ degrees})$
$c^2 = 1010 - 569$
$c^2 = 441$
$c = 21$

7)

14^2 = 9^2 + 6^2 - 2*9*6*cosx
196 = 81 + 36 - 108cosx
196 = 117 - 108cosx
79 = -108cosx
79/-108 = cosx
-.7314=cosx
cos(x) = -.7314
Use the Cos^{-1} button. Locate it!
x = 137

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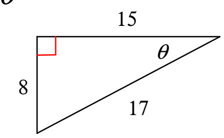
Right Triangle Trig. - Evaluating Trig. Ratios

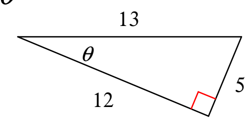
Date________________ Period____

Find the value of the trig function indicated.

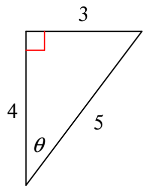
1) sec θ



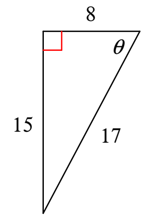
2) sec θ



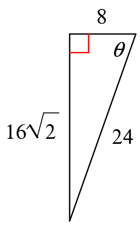
3) cot θ



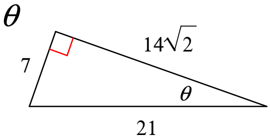
4) csc θ



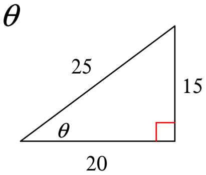
5) csc θ



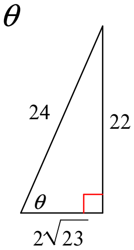
6) cos θ



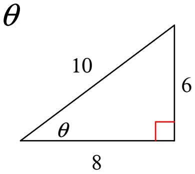
7) cot θ



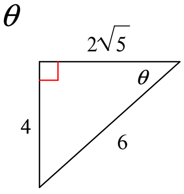
8) tan θ



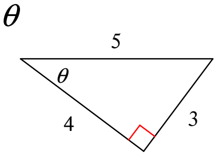
9) tan θ



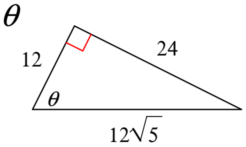
10) cot θ



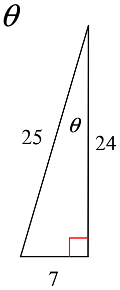
11) tan θ



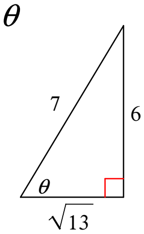
12) cot θ



13) tan θ



14) sin θ



Find the value of each. Round your answers to the nearest ten-thousandth.

15) cos 10°

16) sin 60°

17) csc 21°

18) cos 60°

19) tan 40°

20) csc 59°

21) csc 56°

22) cot 65°

23) tan 10°

24) tan 25°

Find the value of the trig function indicated.

25) Find csc θ if tan $\theta = \frac{3}{4}$

26) Find cot θ if sec $\theta = 2$

27) Find tan θ if sin $\theta = \frac{4}{5}$

28) Find cot θ if sec $\theta = \frac{5}{4}$

29) Find sec θ if sin $\theta = \frac{3\sqrt{13}}{13}$

30) Find cot θ if sin $\theta = \frac{12}{13}$

Critical think questions:

31) Draw a right triangle that has an angle with a tangent of 1.

32) What is the slope of the hypotenuse for #9? How does that compare to tan θ? Why?

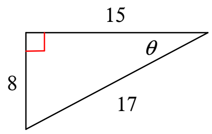
Kuta Software - Infinite Algebra 2 Name_______________

Right Triangle Trig. - Evaluating Trig. Ratios

Date________________ Period____

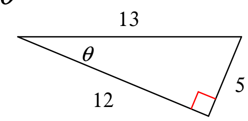
Find the value of the trig function indicated.

1) sec θ



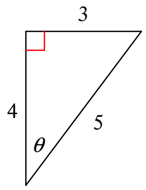
$\frac{17}{15}$

2) sec θ



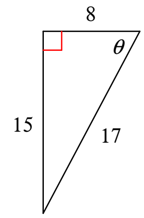
$\frac{13}{12}$

3) cot θ



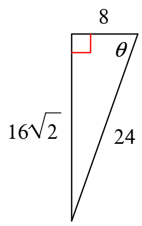
$\frac{4}{3}$

4) csc θ



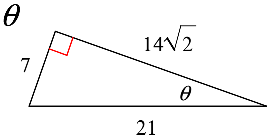
$\frac{17}{15}$

5) csc θ



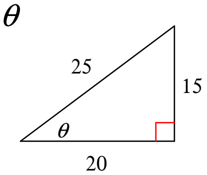
$\frac{3\sqrt{2}}{4}$

6) cos θ



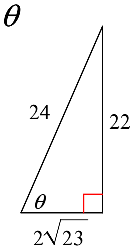
$\frac{2\sqrt{2}}{3}$

7) cot θ



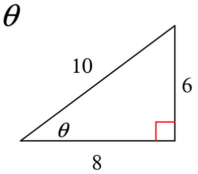
$\frac{4}{3}$

8) tan θ



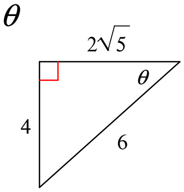
$\frac{11\sqrt{23}}{23}$

9) tan θ



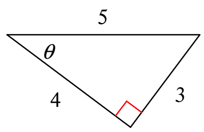
$\frac{3}{4}$

10) cot θ



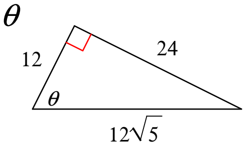
$\frac{\sqrt{5}}{2}$

11) tan θ



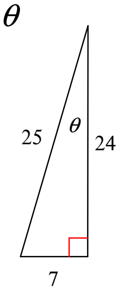
$\frac{3}{4}$

12) cot θ

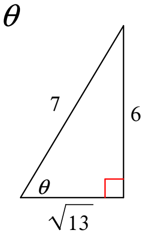


$\frac{1}{2}$

13) tan θ $\frac{7}{24}$



14) sin θ $\frac{6}{7}$



Find the value of each. Round your answers to the nearest ten-thousandth.

15) cos 10°

0.9848

16) sin 60°

0.8660

17) csc 21°

2.7904

18) cos 60°

0.5000

19) tan 40°

0.8391

20) csc 59°

1.1666

21) csc 56°

1.2062

22) cot 65°

0.4663

23) tan 10°

0.1763

24) tan 25°

0.4663

Find the value of the trig function indicated.

25) Find csc θ if tan $\theta = \frac{3}{4}$ $\frac{5}{3}$

26) Find cot θ if sec $\theta = 2$ $\frac{\sqrt{3}}{3}$

27) Find tan θ if sin $\theta = \frac{4}{5}$ $\frac{4}{3}$

28) Find cot θ if sec $\theta = \frac{5}{4}$ $\frac{4}{3}$

29) Find sec θ if sin $\theta = \frac{3\sqrt{13}}{13}$ $\frac{\sqrt{13}}{2}$

30) Find cot θ if sin $\theta = \frac{12}{13}$ $\frac{5}{12}$

Critical think questions:

31) Draw a right triangle that has an angle with a tangent of 1.

Any right isosceles triangle.

32) What is the slope of the hypotenuse for #9? How does that compare to tan θ? Why?

$\frac{3}{4}$ It's the same as tan θ since rise/run = opp/adj

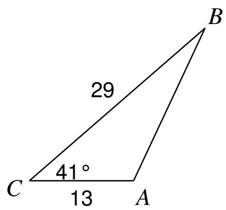
Kuta Software - Infinite Algebra 2 Name________________________

The Law of Cosines

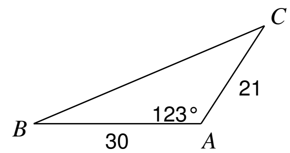
Date________________ Period____

Find each measurement indicated. Round your answers to the nearest tenth.

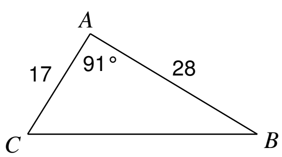
1) Find AB



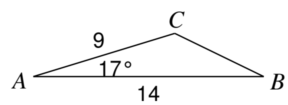
2) Find BC



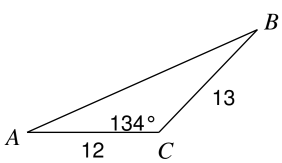
3) Find BC



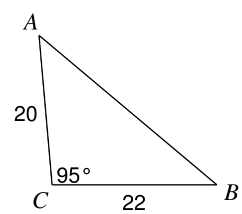
4) Find BC



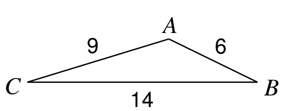
5) Find AB



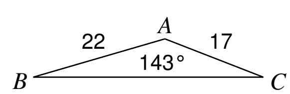
6) Find AB



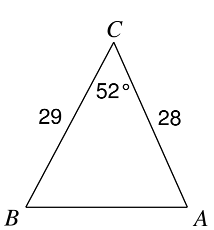
7) Find $m\angle$A



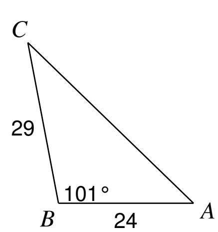
8) Find $m\angle$B



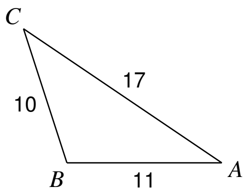
9) Find $m\angle$A



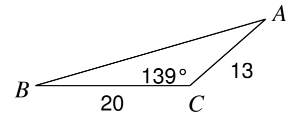
10) Find $m\angle$C



11) Find $m\angle A$

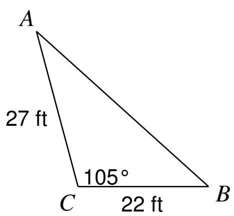


12) Find $m\angle A$

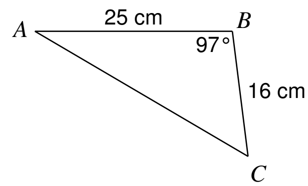


Solve each triangle. Round your answers to the nearest tenth.

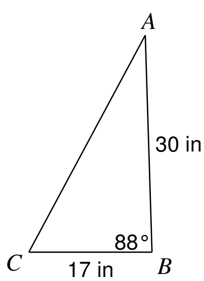
13)



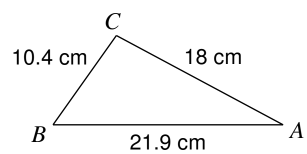
14)



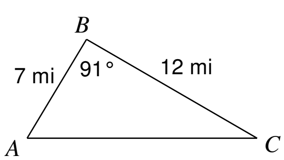
15)



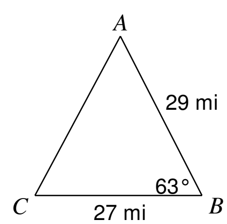
16)



17)



18)



19) In ΔABC, $a = 14$ cm, $b = 9$ cm, $c = 6$ cm

20) In ΔXYZ, $m\angle X = 138°$, $y = 15$ in, $z = 25$ in

21) In ΔQRP, $q = 12$ in, $p = 28$ in, $r = 18$ in

22) In ΔQRP, $p = 28$ km, $q = 17$ km, $r = 15$ km

23) In ΔDEF, $e = 16$ yd, $d = 12$ yd, $f = 17$ yd

24) In ΔRPQ, $p = 18$ mi, $m\angle R = 17°$, $q = 28$ mi

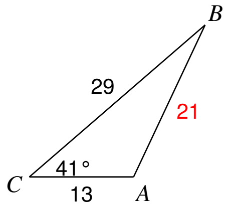
Kuta Software - Infinite Algebra 2 Name___________________________________

The Law of Cosines

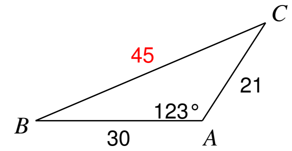
Date____________________ Period____

Find each measurement indicated. Round your answers to the nearest tenth.

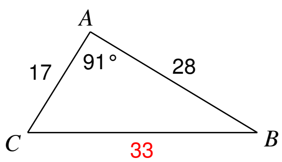
1) Find AB



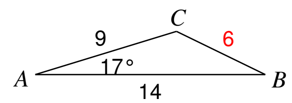
2) Find BC



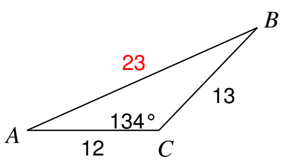
3) Find BC



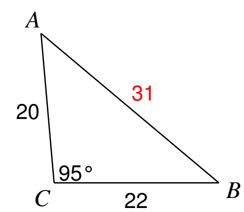
4) Find BC



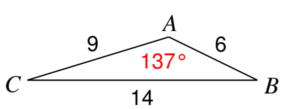
5) Find AB



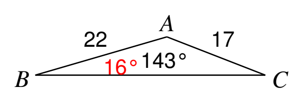
6) Find AB



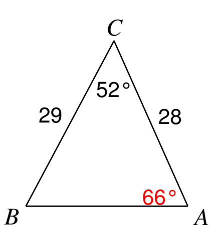
7) Find $m\angle$A



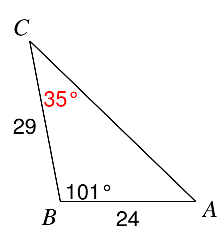
8) Find $m\angle$B



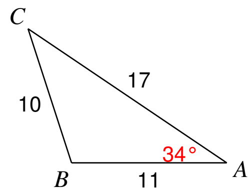
9) Find $m\angle$A



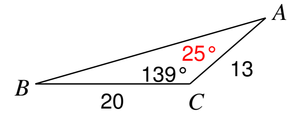
10) Find $m\angle$C



11) Find $m\angle A$

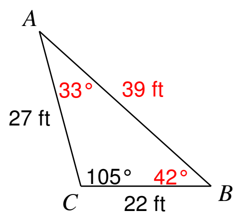


12) Find $m\angle A$



Solve each triangle. Round your answers to the nearest tenth.

13)



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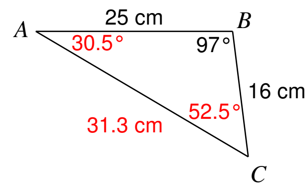

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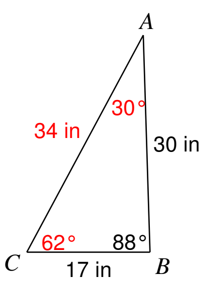

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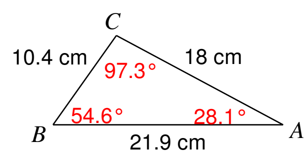

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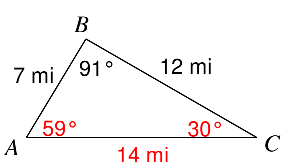

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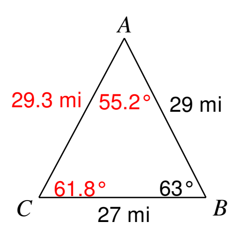


19) In ΔABC, $a = 14$ cm, $b = 9$ cm, $c = 6$ cm

$m\angle A = 137°$, $m\angle B = 26°$, $m\angle C = 17°$

20) In ΔXYZ, $m\angle X = 138°$, $y = 15$ in, $z = 25$ in

$m\angle Y = 15.5°$, $m\angle Z = 26.5°$, $x = 37.5$ in

21) In ΔQRP, $q = 12$ in, $p = 28$ in, $r = 18$ in

$m\angle Q = 17°$, $m\angle R = 26°$, $m\angle P = 137°$

22) In ΔQRP, $p = 28$ km, $q = 17$ km, $r = 15$ km

$m\angle Q = 31°$, $m\angle R = 27°$, $m\angle P = 122°$

23) In ΔDEF, $e = 16$ yd, $d = 12$ yd, $f = 17$ yd

$m\angle D = 42.5°$, $m\angle E = 64.3°$, $m\angle F = 73.2°$

24) In ΔRPQ, $p = 18$ mi, $m\angle R = 17°$, $q = 28$ mi

$m\angle P = 26°$, $m\angle Q = 137°$, $r = 12$ mi

Lesson 18 Summary: April 2nd

Treviso Arithmetic
Imaginary Numbers
Vi Hart Video
Absolute Value Formula for Complex Numbers

Lesson Handouts

Treviso Arithmetic (Cliff Pickover, The Math Book)
Operations with Complex Numbers worksheet (Kuta)
Properties of Complex Numbers worksheet (Kuta)

5 Out Of 4 People Have A Problem Understanding Jokes About Fractions.

WARM UP:

θ

1. What are the three most common trigonometric functions?

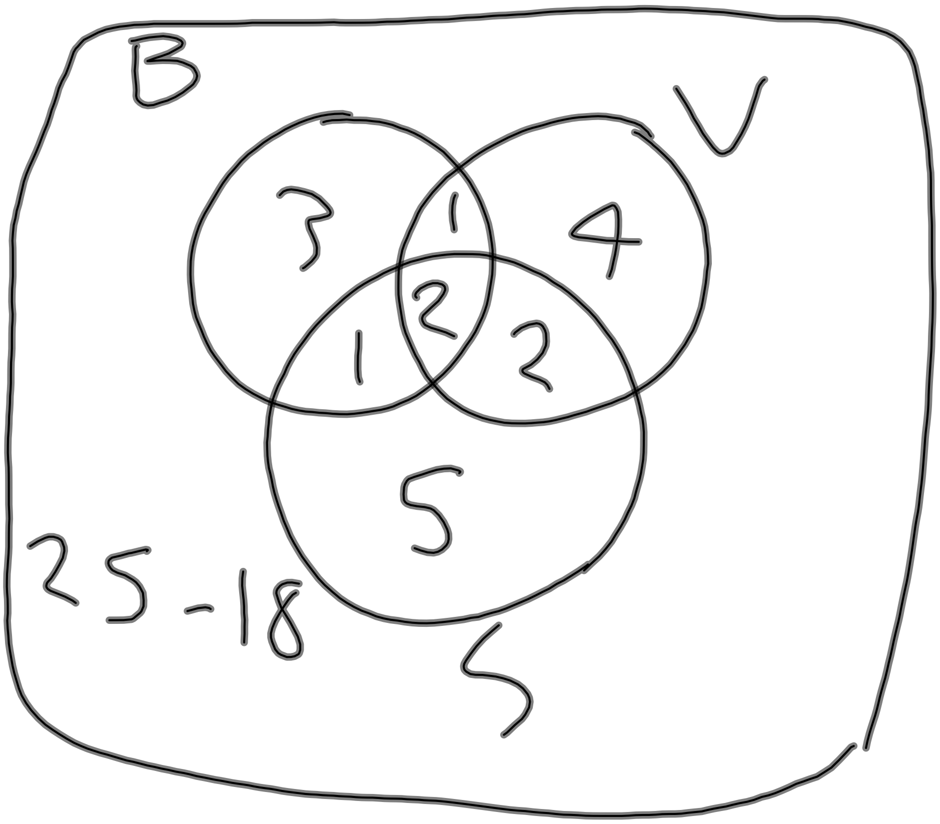
2. How are they defined, what is the mnemonic? SOHCAHTOA

3. Try to locate the 2 methods of measuring angle on your calculator. (one is obvious)

$2\pi = 360$

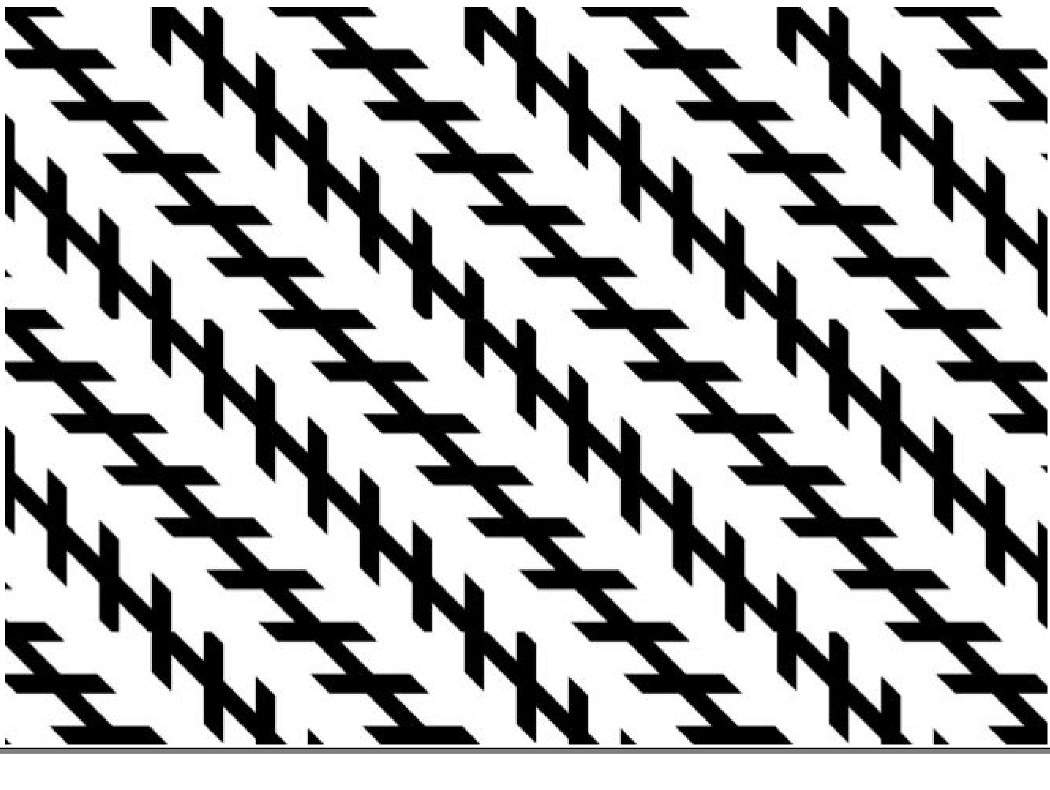
4. Define the Law of Cosines, draw a diagram explaining what each variable stands for.

5. Of the seventh graders at Darmuth middle school, 7 played basketball, 9 played volleyball, 10 played soccer, and 3 played basketball and volleyball, 3 played basketball and soccer, 4 played volleyball and soccer, and 2 played volleyball, basketball and soccer. There are a total of 25 seventh graders that attend Darmuth middle school. How many students played no sports?



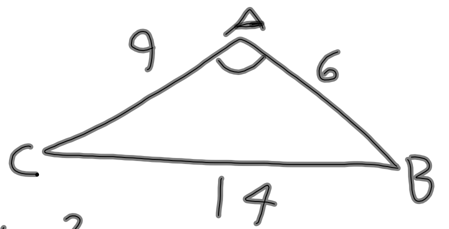
Visual Fun for the Day:

The Zöllner illusion: Despite their appearance, the black lines are parallel.

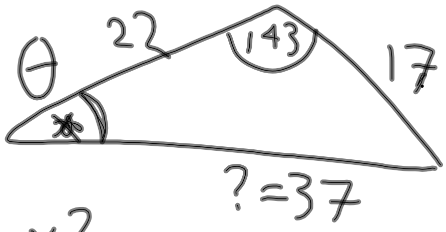


Let's use the Law of Cosines to complete more of the worksheet.

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$



$$14^2 = 9^2 + 6^2 - 2 \cdot 9 \cdot 6 \cdot \cos A$$
$$196 = 117 - 108\cos A$$
$$79 = -108\cos A$$
$$\frac{79}{-108} = \cos A$$
$$A = 137$$



$$x^2 = 22^2 + 17^2 - 2 \cdot 22 \cdot 17 \cdot \cos 143$$
$$x^2 = 773 - -597$$
$$x^2 = 1370$$

$$17^2 = 22^2 + 37^2 - 2 \cdot 22 \cdot 37 \cos\theta$$

TREVISO ARITHMETIC

The Treviso Arithmetic is **the earliest known printed mathematics book in the West**, and one of the first printed European textbooks dealing with a science. It helped to end the monopoly on mathematical knowledge and gave important information to the middle class. The Treviso became one of the first mathematics books that were written for the expansion of human knowledge. It gave opportunity for the common person to learn the art of computation instead of only a privileged few. The Treviso Arithmetic provided an early example of the Hindu-Arabic numeral system and computational algorithms

Three merchants have invested their money as a company. The first is Piero. The other is Polo. The third is Zuanne. Piero put in as his capital 112 ducats. Polo put in as his capital 200 ducats. Zuanne put in as his capital 142 ducats. And at the end of a certain period of time they have realized a profit of 563 ducats. I ask how much each man gets so that no one will be cheated.

454 = TOTAL

25%
44% > 563
31%

$\sqrt{-1} = i$

Imaginary numbers are those that when we square, we get a negative number.

Numbers that contain an "i" are called complex numbers and are of the form a + bi where a and b are real numbers.

The deeper we dive into physics and the nature of reality, the more complex numbers appear.

Let's watch a short video about "i" by the mathemusician Vi Hart.

http://www.youtube.com/watch?v=WU3AlAOCxN0&list=UUOGeU-1Fig3rrDjhm9Zs_wg&index=1

Some basic rules to follow:

- We add imaginary components as we do like terms in algebra.

- If i is the square root of -1, then what would i^2 be? $i \cdot i = i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

- $\sqrt{-4} = \sqrt{4 * -1} = 2\sqrt{-1} = 2i$

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$

The absolute value of a real number is its distance from 0 on the number line. A complex number's absolute value is also a measure of its distance from zero. However, instead of measuring this distance on the number line, a complex number's absolute value is measured from zero on the complex number plane.

Absolute Value Formula:

$|a + bi| = \sqrt{a^2 + b^2}$

1478

Treviso Arithmetic

European arithmetic texts in the fifteenth and sixteenth centuries often presented mathematical word problems related to commerce in order to teach mathematical concepts. The general idea of word problems for students dates back for centuries, and some of the oldest-known word problems were presented in ancient Egypt, China, and India.

Treviso Arithmetic is brimming with word problems, many involving merchants investing money and who wish to avoid being cheated. The book was written in a Venetian dialect and published in 1478 in the town of Treviso, Italy. The unknown author of the book writes, "I have often been asked by certain youths in whom I have much interest, and who look forward to mercantile pursuits, to put into writing the fundamental principles of arithmetic. Therefore, being impelled by my affection for them, and by the value of the subject, I have to the best of my small ability undertaken to satisfy them in some slight degree." He then gives numerous word problems involving merchants with names like Sebastiano and Jacomo who invest their money for gain in a partnership. The book also shows several ways for performing multiplication and includes information from the Fibonacci work ***Liber Abaci*** (1202).

Treviso is particularly significant because it is the earliest-known printed mathematics book in Europe. It also promoted the use of the Hindu Arabic numeral system and computational algorithms. Because commerce of the time began to have a wide international component, prospective businessmen had urgent needs to become facile with mathematics. Today, scholars are fascinated by *Treviso* because it provides a portal into the methods of teaching mathematics in fifteenth-century Europe. Similarly, because the problems involve calculating payment for exchanging goods, fabric cutting, saffron trading, alloy mixtures in coins, currency exchange, and calculating shares of profits derived from partnerships, readers come to understand the contemporary concerns with respect to cheating, usury, and determination of interest charges.

SEE ALSO Rhind Papyrus (1650 B.C.), *Ganita Sara Samgraha* (850), Fibonacci's *Liber Abaci* (1202), and *Sumario Compendioso* (1556).

Merchants weighing their goods in a marketplace, circa 1400, drawn after a fifteenth-century stained-glass window in Chartres Cathedral, France. Treviso Arithmetic, *the earliest-known printed mathematics book in Europe, includes problems involving merchants, investment, and trade.*

Kuta Software - Infinite Algebra 2 Name________________________________

Operations with Complex Numbers

Date________________ Period____

Simplify.

1) $i + 6i$

2) $3 + 4 + 6i$

3) $3i + i$

4) $-8i - 7i$

5) $-1 - 8i - 4 - i$

6) $7 + i + 4 + 4$

7) $-3 + 6i - (-5 - 3i) - 8i$

8) $3 + 3i + 8 - 2i - 7$

9) $4i(-2 - 8i)$

10) $5i \cdot -i$

11) $5i \cdot i \cdot -2i$

12) $-4i \cdot 5i$

13) $(-2 - i)(4 + i)$

14) $(7 - 6i)(-8 + 3i)$

15) $7i \cdot 3i(-8 - 6i)$

16) $(4 - 5i)(4 + i)$

17) $(2 - 4i)(-6 + 4i)$

18) $(-3 + 2i)(-6 - 8i)$

19) $(8 - 6i)(-4 - 4i)$

20) $(1 - 7i)^2$

21) $6(-7 + 6i)(-4 + 2i)$

22) $(-2 - 2i)(-4 - 3i)(7 + 8i)$

23) $5i + 7i \cdot i$

24) $(6i)^3$

25) $6i \cdot -4i + 8$

26) $-6(4 - 6i)$

27) $(8 - 3i)^2$

28) $3 + 7i - 3i - 4$

29) $-3i \cdot 6i - 3(-7 + 6i)$

30) $-6i(8 - 6i)(-8 - 8i)$

Critical thinking questions:

31) How are the following problems different?

Simplify: $(2 + x)(3 - 2x)$
Simplify: $(2 + i)(3 - 2i)$

32) How are the following problems different?

Simplify: $2 + x - (3 - 2x)$
Simplify: $2 + i - (3 - 2i)$

Kuta Software - Infinite Algebra 2 Name________________________________

Operations with Complex Numbers

Date________________ Period____

Simplify.

1) $i + 6i$

$7i$

2) $3 + 4 + 6i$

$7 + 6i$

3) $3i + i$

$4i$

4) $-8i - 7i$

$-15i$

5) $-1 - 8i - 4 - i$

$-5 - 9i$

6) $7 + i + 4 + 4$

$15 + i$

7) $-3 + 6i - (-5 - 3i) - 8i$

$2 + i$

8) $3 + 3i + 8 - 2i - 7$

$4 + i$

9) $4i(-2 - 8i)$

$32 - 8i$

10) $5i \cdot -i$

5

11) $5i \cdot i \cdot -2i$

$10i$

12) $-4i \cdot 5i$

20

13) $(-2 - i)(4 + i)$

$-7 - 6i$

14) $(7 - 6i)(-8 + 3i)$

$-38 + 69i$

15) $7i \cdot 3i(-8 - 6i)$

$168 + 126i$

16) $(4 - 5i)(4 + i)$

$21 - 16i$

17) $(2 - 4i)(-6 + 4i)$

$4 + 32i$

18) $(-3 + 2i)(-6 - 8i)$

$34 + 12i$

19) $(8 - 6i)(-4 - 4i)$

$-56 - 8i$

20) $(1 - 7i)^2$

$-48 - 14i$

21) $6(-7 + 6i)(-4 + 2i)$

$96 - 228i$

22) $(-2 - 2i)(-4 - 3i)(7 + 8i)$

$-98 + 114i$

23) $5i + 7i \cdot i$

$-7 + 5i$

24) $(6i)^3$

$-216i$

25) $6i \cdot -4i + 8$

32

26) $-6(4 - 6i)$

$-24 + 36i$

27) $(8 - 3i)^2$

$55 - 48i$

28) $3 + 7i - 3i - 4$

$-1 + 4i$

29) $-3i \cdot 6i - 3(-7 + 6i)$

$39 - 18i$

30) $-6i(8 - 6i)(-8 - 8i)$

$-96 + 672i$

Critical thinking questions:

31) How are the following problems different?

Simplify: $(2 + x)(3 - 2x)$
Simplify: $(2 + i)(3 - 2i)$

$i^2 = -1$ so it leads to a few more steps

32) How are the following problems different?

Simplify: $2 + x - (3 - 2x)$
Simplify: $2 + i - (3 - 2i)$

There is no difference.

Kuta Software - Infinite Algebra 2 Name________________________

Properties of Complex Numbers

Date________________ Period____

Find the absolute value of each complex number.

1) $|7 - i|$

2) $|-5 - 5i|$

3) $|-2 + 4i|$

4) $|3 - 6i|$

5) $|10 - 2i|$

6) $|-4 - 8i|$

7) $|-4 - 3i|$

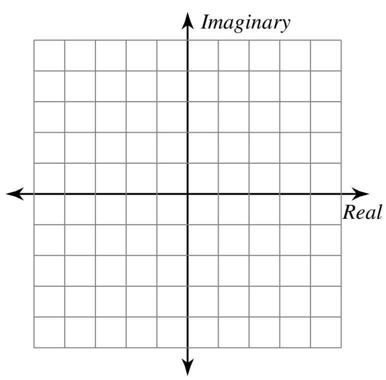
8) $|8 - 3i|$

9) $|1 - 8i|$

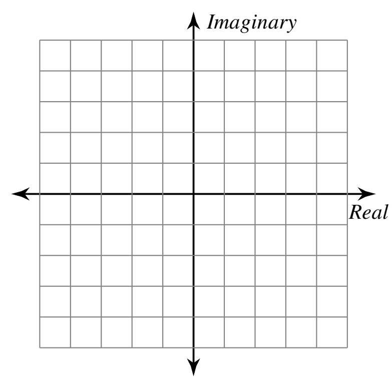
10) $|-4 + 10i|$

Graph each number in the complex plane.

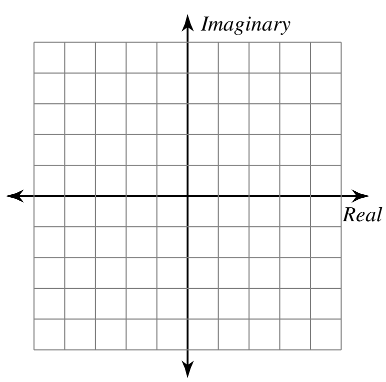
11) $-3 + 4i$



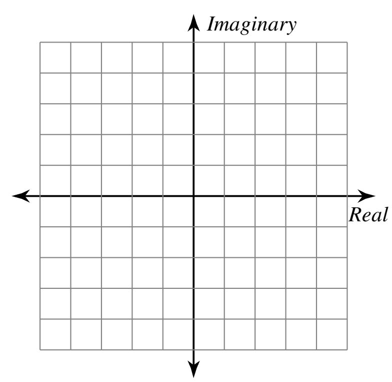
12) $-1 + 5i$



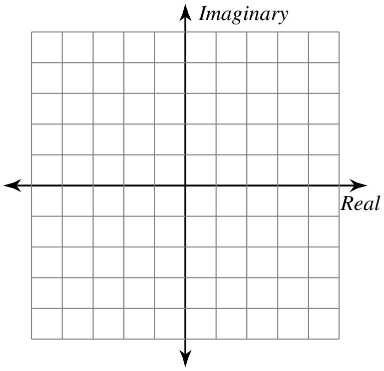
13) $-1 - 4i$



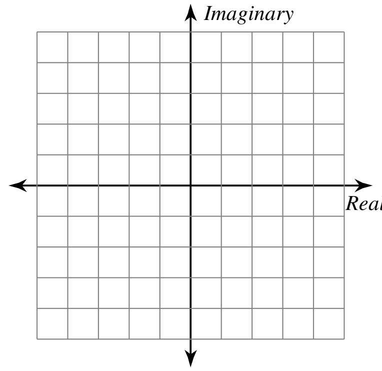
14) $4 + 4i$



15) $-3 + 5i$

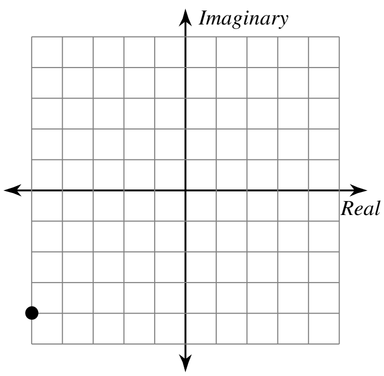


16) $2 + 4i$



Identify each complex number graphed.

17)



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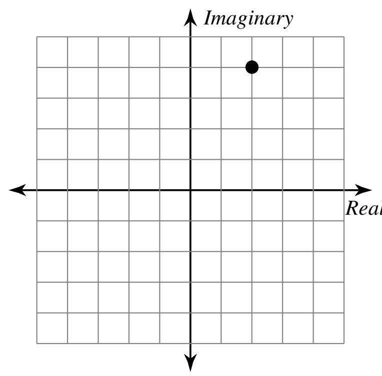

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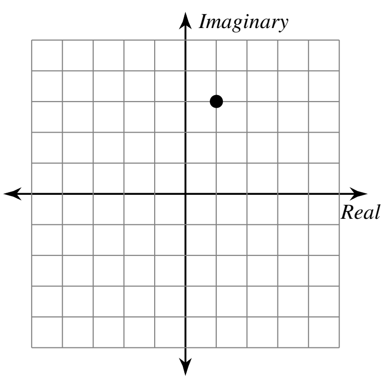

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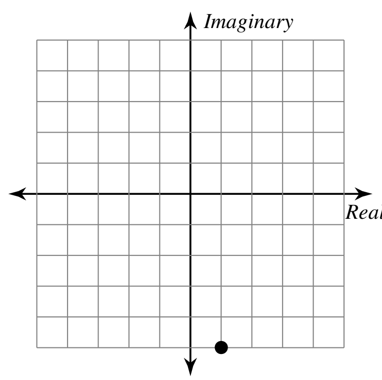
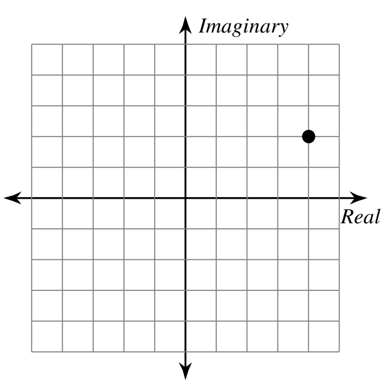


21)



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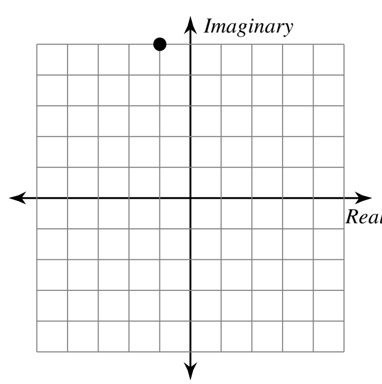


Kuta Software - Infinite Algebra 2 Name________________________________

Properties of Complex Numbers

Date________________ Period____

Find the absolute value of each complex number.

1) $|7 - i|$

$5\sqrt{2}$

2) $|-5 - 5i|$

$5\sqrt{2}$

3) $|-2 + 4i|$

$2\sqrt{5}$

4) $|3 - 6i|$

$3\sqrt{5}$

5) $|10 - 2i|$

$2\sqrt{26}$

6) $|-4 - 8i|$

$4\sqrt{5}$

7) $|-4 - 3i|$

5

8) $|8 - 3i|$

$\sqrt{73}$

9) $|1 - 8i|$

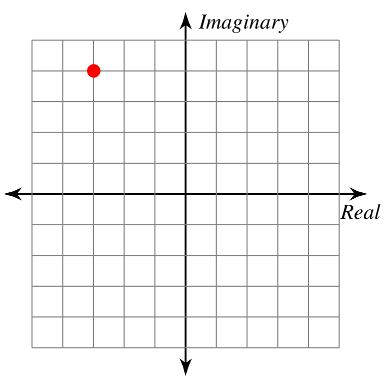
$\sqrt{65}$

10) $|-4 + 10i|$

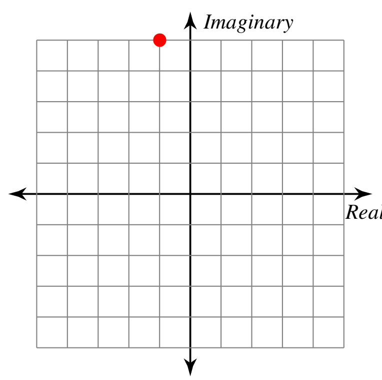
$2\sqrt{29}$

Graph each number in the complex plane.

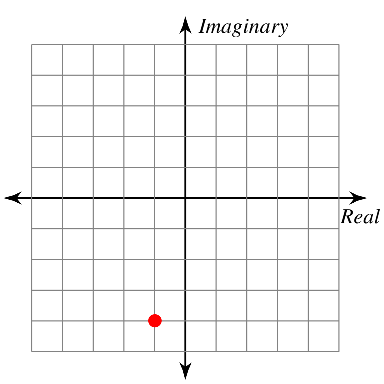
11) $-3 + 4i$



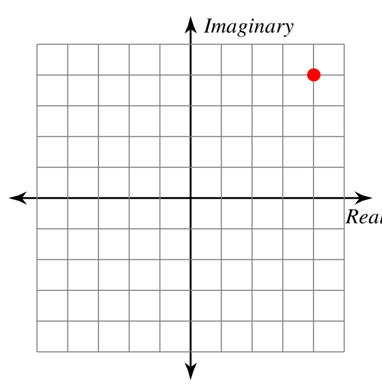
12) $-1 + 5i$



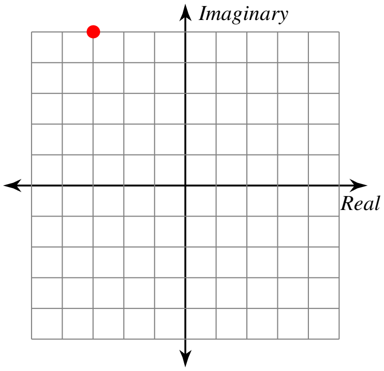
13) $-1 - 4i$



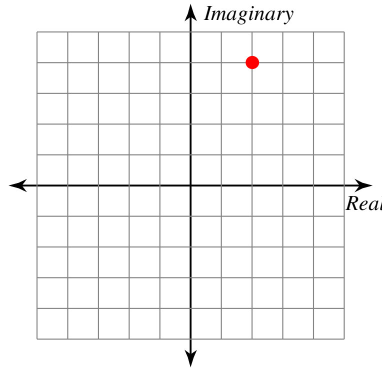
14) $4 + 4i$



15) $-3 + 5i$

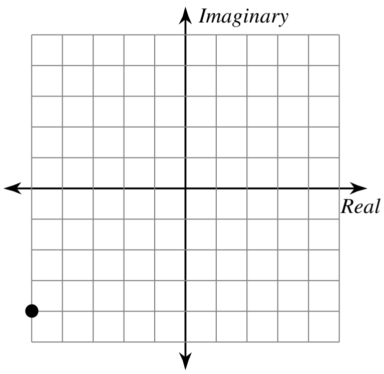


16) $2 + 4i$



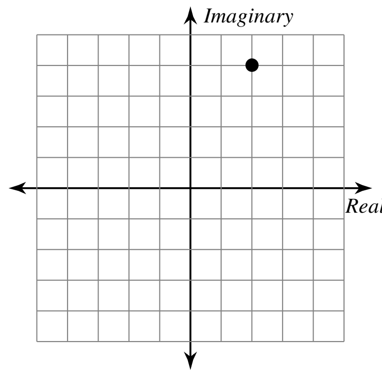
Identify each complex number graphed.

17)



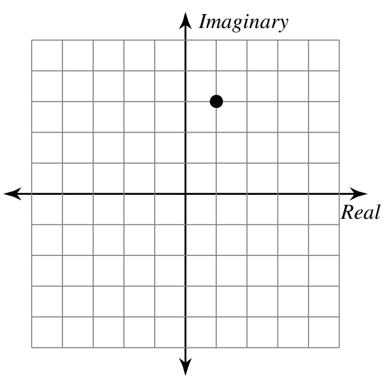
$-5 - 4i$

18)



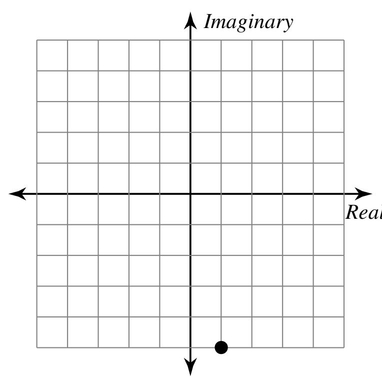
$2 + 4i$

19)



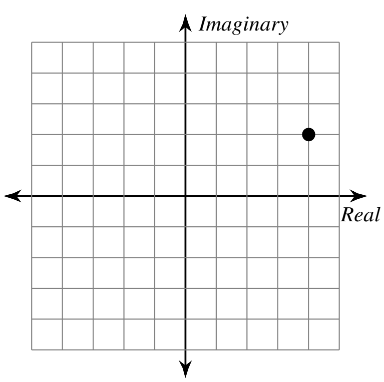
$1 + 3i$

20)



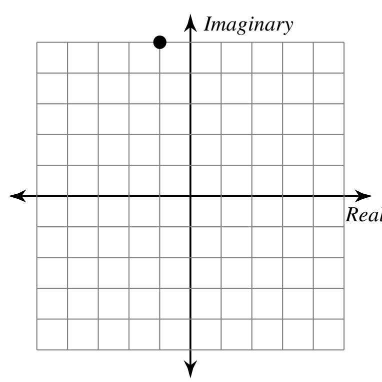
$1 - 5i$

21)



$4 + 2i$

22)



$-1 + 5i$

Lesson 19 Summary: April 4th
Word Play
Round 2: Imaginary Numbers

Lesson Handouts
Properties of Complex Numbers worksheet (Kuta)

Logic is invincible because in order to combat logic it is necessary to use logic.
— Pierre Boutroux

Mathematics is the handwriting on the human consciousness of the very Spirit of Life itself.
— Claude Bragdon

Ask not what's inside your head but what your head's inside of.
— William Mace

Mar 12-1:33 PM

WARM UP:

1. Use the Law of Cosines to solve a triangle (find all 3 angle measures) given the 3 side lengths are known to be: a = 2.969, b = 5, c=4.864.

$\sqrt{-1} = i$ $12i$

2. Simplify $\sqrt{-144}$ and $\sqrt{-200}$ using imaginary numbers.

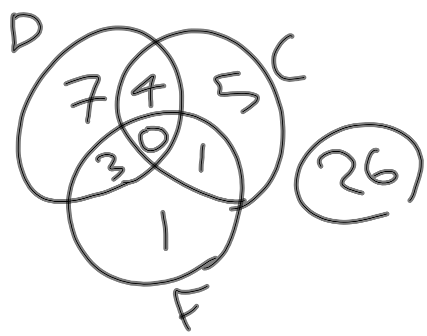
$10\sqrt{2}$

3. John looks at Ann, but Ann is looking at Harry. John is married but Harry is not. Is a married person looking at an unmarried person?

J → A → H
m   um

4. A veterinarian surveys 26 of his patrons. He discovers that 14 have dogs, 10 have cats, and 5 have fish. Four have dogs and cats, 3 have dogs and fish, and one has a cat and fish. If no one has all three kinds of pets, how many patrons have none of these pets?

Feb 6-9:21 AM



Apr 4-1:22 PM

$$\sqrt{200} =$$

$$\sqrt{10 \cdot 10 \cdot 2} =$$

$$10\sqrt{2}$$

Apr 4-1:16 PM

Example 2. Use the Law of Cosines to Solve SSS Triangle.

Solve the triangle: **a** = 2.969, **b** = 5, **c**=4.864.
This is **SSS** case: three sides of the triangle are given.
Figure 4 illustrates the triangle we are going to solve.

First, find the angle α using **Law of Cosines** with known sides **a**, **b** and **c**.
Since $a^2=b^2+c^2-2\cdot b\cdot c\cdot\cos(\alpha)$ (**Law of Cosines**),
you have $\cos(\alpha)=\frac{b^2+c^2-a^2}{2\cdot b\cdot c}=\frac{5^2+4.864^2-2.969^2}{2\cdot 5\cdot 4.864}=0.819$.
Hence, $\alpha=\arccos(0.819)=35°$.

Next, find the angle β by the similar way:
$b^2=a^2+c^2-2\cdot a\cdot c\cdot\cos(\beta)$, $\cos(\beta)=\frac{a^2+c^2-b^2}{2\cdot a\cdot c}=\frac{2.969^2+4.864^2-5^2}{2\cdot 2.969\cdot 4.864}=0.259$, and $\beta=75°$.

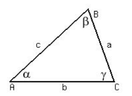


Figure 4. Example 2

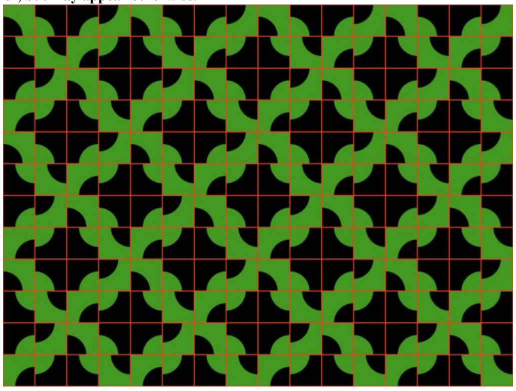
Now, calculate the third angle γ of the triangle, having known other two, α and β:
$\gamma = 180° - (\alpha+\beta) = 180° - (35°+75°) = 180° - 110° = 70°$.

The solution is unique in this case.

Apr 4-11:48 AM

Visual Fun for the Day:

The red-orange lines are strictly horizontal/vertical, and fully parallel/perpendicular to each other, but may appear otherwise.



Feb 11-12:17 PM

A LITTLE NUMBER SENSE WITH WORD PLAY

An estimate of how many English language words the average person knows is typically in the realm of 12,000 - 20,000 words, varying with the level of education achieved. Some say college/university graduates use upwards of **20-25,000 word**s. These estimates apply to natives of the English language. Shakespeare actively used more than 30,000 words in his written works, and his entire vocabulary has been estimated at approximately 66,000 words.
The English Language passed the **Million Word** threshold on June 10, 2009. The Millionth Word was the controversial 'Web 2.0′. Currently there is a new word created every 98 minutes or about 14.7 words per day.

1. Calculate what you think your percentage is of words you know, if you consider a million total English words.

2. What is the advantage to verbosity? Does it matter?

Apr 4-12:08 PM

Let's finish the complex number worksheets. In order to, lets first consider the following:
The absolute value of a real number is its distance from 0 on the number line. A complex number's absolute value is also a measure of its distance from zero. However, instead of measuring this distance on the number line, a complex number's absolute value is measured from zero on the complex number plane.

Absolute Value Formula:

$|a+bi| = \sqrt{a^2+b^2}$

$\sqrt{7^2+(-1)^2} =$

$\sqrt{49+1} = \sqrt{50}$

$5\sqrt{2}$

Apr 2-12:24 PM

$5i \cdot i - 2i =$

$-10 \cdot i^2 \cdot i =$

$10i$

Apr 4-1:31 PM

13) $-8-2i-4i-i^2$

$(2-i)(4+i) \quad -7-6i$

Apr 4-1:34 PM

$21i^2(-8-6i)=$

$-21(-8-6i)=$

$168+126i$

Apr 4-1:43 PM

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2$

$= \sqrt{-1} \cdot \sqrt{-1}$

$= -1$

$\sqrt{13} \cdot \sqrt{13} =$

$\sqrt{169} =$

13

Apr 4-1:45 PM

16)

1 $= 16 + 4i - 20i - 5i^2$

2 $= 16 - 16i + 5$

3 $= 21 - 16i$

Apr 4-1:49 PM

$(1 - 7i)^2 =$ $5x^2$

$(1-7i)(1-7i)$

$1 - 7i - 7i + 49i^2$

$1 - 14i - 49$

$-48 - 14i$

Apr 4-2:01 PM

<u>Lesson 20 Summary: April 9th</u>
Probability Problem
Logarithms and Napier

<u>Lesson Handouts</u>
The Meaning of Logarithms worksheet (Kuta)
Properties of Logarithms (Kuta)

The (wo)man ignorant of mathematics will be increasingly limited in his (her) grasp of the main forces of civilization.

WARM UP:

1.If $i^0 = 1$, $i^1 = i$ and $i^2 = -1$ and $i^3 = -i$ and $i^4 = 1$, then what is i^5? How about i^6?

2. There is only one time in your life when you're twice as old as your child. When is that?

If two fair dice are rolled, find the probabilities of the following results.

5. A sum of 8, given that the sum is greater than 7

6. A sum of 6, given that the roll was a "double" (two identical numbers)

7. A double, given that the sum was 9

8. A double, given that the sum was 8.

PROBABILITY PROBLEM (TOUGH)

Blood Pressure A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.

(ii) 22% have low blood pressure.

(iii) 15% have an irregular heartbeat.

(iv) Of those with an irregular heartbeat, one-third have high blood pressure.

(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

What portion of the patients selected have a regular heartbeat and low blood pressure? Choose one of the following. (*Hint:*

	HIGH	LOW	NORMAL	TOTALS
REGULAR	? 7th	? 6th	? 4th	85
IRREGULAR	? 1st	? 5th	? 3rd	15
TOTALS	14	22	? 2nd	100

	HIGH	LOW	NORMAL	TOTALS
REGULAR	9	20	56	85
IRREGULAR	5	2	8	15
TOTALS	14	22	64	100

ANSWER = 20%

Visual Fun for the Day:

Let's finish the complex number worksheets. In order to, lets first consider the following: Absolute Value Formula: $|a+bi| = \sqrt{a^2 + b^2}$

$= -6i(-64 - 64i + 48i + 48i^2)$

$= -6i(-112 - 16i)$

$= 672i + 96i^2$

$= -96 + 672i$

31) $6 - x - 2x^2$

$8 - i$

---

32) $-1 + 3x$

$-1 + 3i$

$\sqrt{50} = \sqrt{5 \cdot 5 \cdot 2}$

$= 5\sqrt{2}$

$2\sqrt{5}$

HISTORY

Just like Trigonometry tends to get a bad rap, so too does the notions of **Logarithms**. Logarithms were invented (discovered?) by the Scottish mathematician John Napier in his 1614 book, "*A Description of the Marvelous Rules of Logarithms*".

Countless breakthroughs in science and engineering have been made possible by making difficult calculations through logarithms, they are used to describe pH levels in chemistry and the Richter scale in earthquake measurement.

Tables of logarithms were a tremendous help in the areas of surveying and navigation until modern calculators and computers were up to the task.

A logarithm is a sort of exponent, where we refer to the log of base b of a number x.

So, the $\log_b(x)$ is equal to the exponent y that satisfies the equation $x = b^y$.

CONSIDER how $3^5 = 243$ is rather straightforward.

Well, we can also express this relationship logarithmically as $\log_3(243)=5$

Logs, like exponents, can have any base. However, the two most common are base 10 and base e (2.71...). On your calculator, the LOG button is base 10, and the LN button is base e.

Logarithms allows us to know ***how many of one number we multiply to get another number***.

For example, how many 2s do we multiply to get 8?

Well, since the answer is obviously 3, we can say that the logarithm base 2 of 8 is 3.

This is written as $\log_2(8) = 3$.

In a way, the logarithm tells you what the exponent is.

Think of logarithms as what exponent do we need for one number to become another number.

SOME BASIC LOG PROBLEMS:

What is $\log_4(256)$? $= 4$ $\quad 4^x = 256$

What is $\log_3(729)$? $= 6$

Write $1{,}024 = 2^{10}$ in logarithmic form. $\log_2(1024) = 10$

Write $\log_3(2{,}187) = 7$ in exponential form.

$\log_4\left(\frac{1}{16}\right) = ?$ $\quad 3^7 = 2187$

$\log_{10}(x) = 3$

$\log_e(7.389) = X$

$\log_5(.008) = ?$

Kuta Software - Infinite Algebra 2 Name____________________

The Meaning Of Logarithms

Date________________ Period____

Rewrite each equation in exponential form.

1) $\log_6 36 = 2$

2) $\log_{289} 17 = \frac{1}{2}$

3) $\log_{14} \frac{1}{196} = -2$

4) $\log_3 81 = 4$

Rewrite each equation in logarithmic form.

5) $64^{\frac{1}{2}} = 8$

6) $12^2 = 144$

7) $9^{-2} = \frac{1}{81}$

8) $\left(\frac{1}{12}\right)^2 = \frac{1}{144}$

Rewrite each equation in exponential form.

9) $\log_u \frac{15}{16} = v$

10) $\log_v u = 4$

11) $\log_{\frac{7}{4}} x = y$

12) $\log_2 v = u$

13) $\log_u v = -16$

14) $\log_y x = -8$

Rewrite each equation in logarithmic form.

15) $u^{-14} = v$

16) $8^b = a$

17) $\left(\frac{1}{5}\right)^{x} = y$

18) $6^{y} = x$

19) $9^{y} = x$

20) $b^{a} = 123$

Evaluate each expression.

21) $\log_{4} 64$

22) $\log_{6} 216$

23) $\log_{4} 16$

24) $\log_{3} \frac{1}{243}$

25) $\log_{5} 125$

26) $\log_{2} 4$

27) $\log_{343} 7$

28) $\log_{2} 16$

29) $\log_{64} 4$

30) $\log_{6} \frac{1}{216}$

Simplify each expression.

31) $12^{\log_{12} 144}$

32) $5^{\log_{5} 17}$

33) $x^{\log_{x} 72}$

34) $9^{\log_{3} 20}$

Name___________________________________

The Meaning Of Logarithms

Date____________________ Period____

Rewrite each equation in exponential form.

1) $\log_6 36 = 2$

$6^2 = 36$

2) $\log_{289} 17 = \frac{1}{2}$

$289^{\frac{1}{2}} = 17$

3) $\log_{14} \frac{1}{196} = -2$

$14^{-2} = \frac{1}{196}$

4) $\log_3 81 = 4$

$3^4 = 81$

Rewrite each equation in logarithmic form.

5) $64^{\frac{1}{2}} = 8$

$\log_{64} 8 = \frac{1}{2}$

6) $12^2 = 144$

$\log_{12} 144 = 2$

7) $9^{-2} = \frac{1}{81}$

$\log_9 \frac{1}{81} = -2$

8) $\left(\frac{1}{12}\right)^2 = \frac{1}{144}$

$\log_{\frac{1}{12}} \frac{1}{144} = 2$

Rewrite each equation in exponential form.

9) $\log_u \frac{15}{16} = v$

$u^v = \frac{15}{16}$

10) $\log_v u = 4$

$v^4 = u$

11) $\log_{\frac{7}{4}} x = y$

$\left(\frac{7}{4}\right)^y = x$

12) $\log_2 v = u$

$2^u = v$

13) $\log_u v = -16$

$u^{-16} = v$

14) $\log_y x = -8$

$y^{-8} = x$

Rewrite each equation in logarithmic form.

15) $u^{-14} = v$

$\log_u v = -14$

16) $8^b = a$

$\log_8 a = b$

17) $\left(\frac{1}{5}\right)^x = y$

$\log_{\frac{1}{5}} y = x$

18) $6^y = x$

$\log_6 x = y$

19) $9^y = x$

$\log_9 x = y$

20) $b^a = 123$

$\log_b 123 = a$

Evaluate each expression.

21) $\log_4 64$

3

22) $\log_6 216$

3

23) $\log_4 16$

2

24) $\log_3 \frac{1}{243}$

-5

25) $\log_5 125$

3

26) $\log_2 4$

2

27) $\log_{343} 7$

$\frac{1}{3}$

28) $\log_2 16$

4

29) $\log_{64} 4$

$\frac{1}{3}$

30) $\log_6 \frac{1}{216}$

-3

Simplify each expression.

31) $12^{\log_{12} 144}$

144

32) $5^{\log_5 17}$

17

33) $x^{\log_x 72}$

72

34) $9^{\log_3 20}$

400

 Name________________________

Properties of Logarithms

Date________________ Period____

Expand each logarithm.

1) $\log (6 \cdot 11)$

2) $\log (5 \cdot 3)$

3) $\log \left(\frac{6}{11}\right)^5$

4) $\log \left(3 \cdot 2^3\right)$

5) $\log \frac{2^4}{5}$

6) $\log \left(\frac{6}{5}\right)^6$

7) $\log \frac{x}{y^6}$

8) $\log (a \cdot b)^2$

9) $\log \frac{u^4}{v}$

10) $\log \frac{x}{y^5}$

11) $\log \sqrt[3]{x \cdot y \cdot z}$

12) $\log \left(x \cdot y \cdot z^2\right)$

Condense each expression to a single logarithm.

13) $\log 3 - \log 8$

14) $\frac{\log 6}{3}$

15) $4\log 3 - 4\log 8$

16) $\log 2 + \log 11 + \log 7$

17) $\log 7 - 2\log 12$

18) $\frac{2\log 7}{3}$

19) $6\log_3 u + 6\log_3 v$

20) $\ln x - 4\ln y$

21) $\log_4 u - 6\log_4 v$

22) $\log_3 u - 5\log_3 v$

23) $20\log_6 u + 5\log_6 v$

24) $4\log_3 u - 20\log_3 v$

Critical thinking questions:

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

26) $\log x \cdot \log 2$

Kuta Software - Infinite Algebra 2 Name___________________

Properties of Logarithms

Date________________ Period____

Expand each logarithm.

1) $\log (6 \cdot 11)$

$\log 6 + \log 11$

2) $\log (5 \cdot 3)$

$\log 5 + \log 3$

3) $\log \left(\frac{6}{11}\right)^5$

$5\log 6 - 5\log 11$

4) $\log (3 \cdot 2^3)$

$\log 3 + 3\log 2$

5) $\log \frac{2^4}{5}$

$4\log 2 - \log 5$

6) $\log \left(\frac{6}{5}\right)^6$

$6\log 6 - 6\log 5$

7) $\log \frac{x}{y^6}$

$\log x - 6\log y$

8) $\log (a \cdot b)^2$

$2\log a + 2\log b$

9) $\log \frac{u^4}{v}$

$4\log u - \log v$

10) $\log \frac{x}{y^5}$

$\log x - 5\log y$

11) $\log \sqrt[3]{x \cdot y \cdot z}$

$\frac{\log x}{3} + \frac{\log y}{3} + \frac{\log z}{3}$

12) $\log (x \cdot y \cdot z^2)$

$\log x + \log y + 2\log z$

Condense each expression to a single logarithm.

13) $\log 3 - \log 8$

$\log \frac{3}{8}$

14) $\frac{\log 6}{3}$

$\log \sqrt[3]{6}$

15) $4\log 3 - 4\log 8$

$\log \frac{3^4}{8^4}$

16) $\log 2 + \log 11 + \log 7$

$\log 154$

17) $\log 7 - 2\log 12$

$\log \frac{7}{12^2}$

18) $\frac{2\log 7}{3}$

$\log \sqrt[3]{7^2}$

19) $6\log_3 u + 6\log_3 v$

$\log_3 (v^6 u^6)$

20) $\ln x - 4\ln y$

$\ln \frac{x}{y^4}$

21) $\log_4 u - 6\log_4 v$

$\log_4 \frac{u}{v^6}$

22) $\log_3 u - 5\log_3 v$

$\log_3 \frac{u}{v^5}$

23) $20\log_6 u + 5\log_6 v$

$\log_6 (v^5 u^{20})$

24) $4\log_3 u - 20\log_3 v$

$\log_3 \frac{u^4}{v^{20}}$

Critical thinking questions:

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

$\log \frac{4x^2}{75y^2}$

26) $\log x \cdot \log 2$

Can't be simplified.

Lesson 21 Summary: April 11th
Round 2: Logarithms
Ciphers

Lesson Handouts
Facebook-PEMDAS Article (Slate)
"Two Children Problem" Article (Science Niblets)

"Life is good for only two things, discovering mathematics and teaching mathematics."
Simeon Poisson

Mar 12-1:33 PM

WARM UP: BB, BG, GB $\frac{1}{3}$

1. Mr. Smith has two children. At least one of them is a boy. What is the probability that both are boys?

2. **Working Women** A survey has shown that 52% of the women in a certain community work outside the home. Of these women, 64% are married, while 86% of the women who do not work outside the home are married. Find the probabilities that a woman in that community can be categorized as follows.

   a. Married

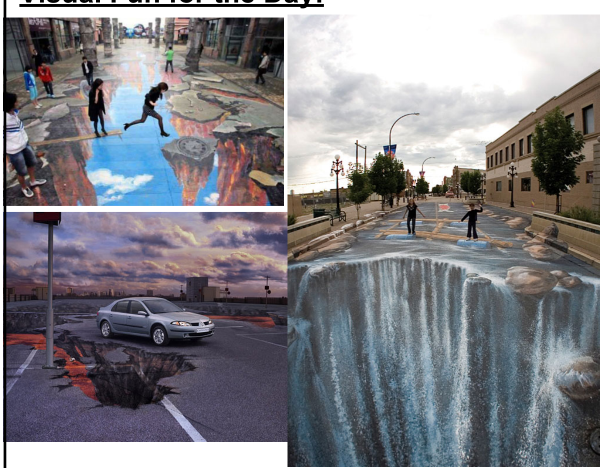
   b. A single woman working outside the home

3. What are the two most common bases used with logarithms? Use your calculator to determine these two logs of 42.

4. $\log_3(.00137) = ?$

Feb 6-9:21 AM

Visual Fun for the Day:



Feb 11-12:17 PM

MORE WITH LOGARITHMS:
When we have a log with a base other than 10 or e (the buttons LOG and LN on a calculator), we can use the **Change of Base formula** to get the answer.

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Let's say we have $\log_3(1/243)$. Using change of base, we can consider this also as: $\log_{anybase}(1/243)/ \log_{anybase}(3)$ = answer.

Logarithms have rules, just like exponents.

Here are a few key properties:

$\log_a(mn) = \log_a m + \log_a n$ *The log of a multiplication is the sum of the logs.*

$\log_a(m/n) = \log_a m - \log_a n$ *The log of a division is the difference of the logs.*

$\log_a(1/n) = -\log_a n$ *Follows from previous "division" rule, because loga(1) = 0.*

$\log_a(m^r) = r (\log_a m)$ *The log of m with an exponent r is r times the log of m.*

Apr 4-11:32 AM

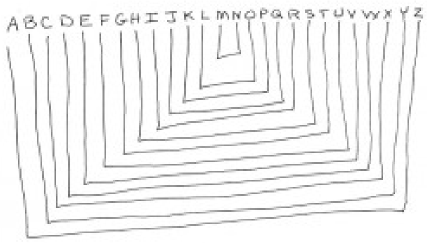
Complete the next page of the Logarithm packet.

$$\log x^{1/3} \cdot y^{1/3} \cdot z^{1/3}$$

$$\frac{\log x}{3} + \frac{\log y}{3} + \frac{\log z}{3}$$

Apr 11-1:33 PM

What is this?

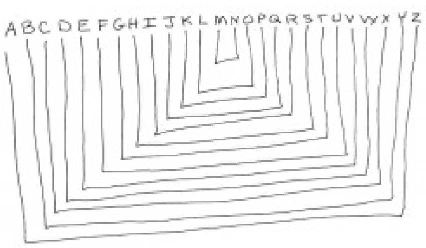


It is an example of a simple **cipher**. A cipher is an algorithm for performing encryption or decryption. Ciphers allow us to hide messages and protect data.

Apr 9-12:39 PM

DECRYPT:

SVOOL, SLD ZIV BLF?



HELLO, HOW ARE YOU?

Apr 11-11:34 AM

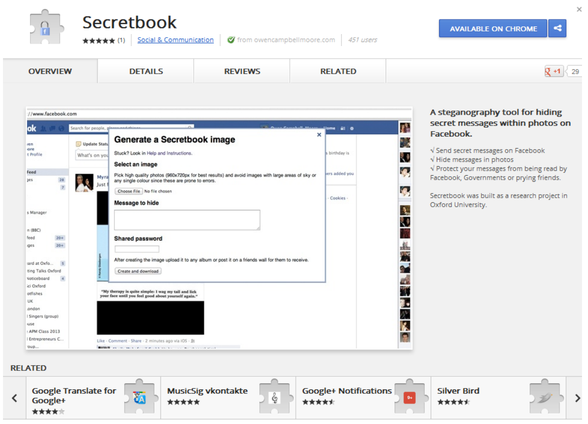
CIPHERS: WHEN DOG =CAT

The first printed book on cryptography, **Polygraphiae Libri Sex, posthumously by Johannes Trithemius in 1518**, was a list of word substitutions. The very interesting thing about this book is that he worded everything so that the encrypted letter would read like a prayer! For example, "CA" as "Conditor clemens", literally "Merciful creator", disguising it effortlessly as a prayer. Ironically enough, some of Trithemius's books were banned by the Church for being "black magic" in 1601, when in reality, they were simply code books!

Now there are many more kinds of ciphers and codes than can be imagined. It is really an entire course of study unto itself. At its most sophisticated, it has ties to national security, espionage and other military-type applications.

Apr 11-10:39 AM

SIMPLE IRL CIPHER:



Apr 11-10:40 AM

Create a Prime Cipher:

1. Determine the first 26 primes. (Sieve of Erastothenes anyone?)

2. Match each letters sequence in the alphabet (C=3) with its corresponding prime. (C=5)

3. Decrypt the following messages using the prime cipher:
41.2.71.19 23.67 5.47.47.37 AND 41.47.43.71.5.47

4. Encrypt an original message using the prime cipher with the answer.

ALPHABET:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
FIRST 26 PRIMES:
2(1) | 3 | 5 | 7 | 11 (5) | 13 | 17 | 19 | 23 | 29 (10) | 31 | 37 | 41 | 43 | 47(15) |
53 | 59 | 61 | 67 | 71 (20) | 73 | 79 | 83 | 89 | 97 (25) | 101
ANSWERS:
(13,2,20,8) (9,19) (3,15,15,12) = 41.2.71.19 23.67 5.47.47.37 = MATH IS COOL
(13,15,14,20,3,15) = 41.47.43.71.5.47=MONTCO

Apr 11-12:03 PM

If Math Is Universal, Why Can't It Answer a Stupid Facebook Problem?

By Tara Haelle | Posted Tuesday, March 12, 2013, at 1:04 PM
| Posted Tuesday, March 12, 2013, at 1:04 PM

Slate.com

What Is the Answer to That Stupid Math Problem on Facebook?

And why are people so riled up about it?

Screenshot courtesy of Facebook

Perhaps you've seen the problem on Facebook or another forum:

6 ÷ 2(1+2) = ?

It's one of several similar math problems popping up on social networks recently. Perhaps you, too, thought, "Duh! That's easy," and then, as I did, became embroiled in an epically long comment thread while your blood pressure steadily rose because you could not possibly understand why the others doing this problem could not get the right answer.

Perhaps, if you're a nerd like me, or you teach math as I do, you even fell asleep thinking about this problem, baffled and frustrated about why you were unable to convince intelligent, educated friends that your calculation of this deceptively simple problem was accurate.

So, did you get 1 or 9? We'll get to the "correct" answer in a moment.

But first, why do we get so riled up about these problems? People don't usually get into fistfights at the bar over arithmetic, but these math threads are spectacularly vitriolic. A couple of factors are at work in these math debates, according to Robert Glenn Howard, a social psychologist at the University of Wisconsin–Madison who specializes in Internet communication and folklore.

For one thing, the whole point of Facebook and other forums is to provide a place for discourse and debate. Yes, there are your cousin's new-baby pictures, and the opportunity to stalk a crush, but really, people go to social sites to say stuff. And argue about it. "People are already primed to engage in pretty intense deliberations, and that can bleed over into the way they play games," Howard says.

And that's exactly what these problems are: games. "Humans have used riddles as a form of play since ancient times," Howard says. "And sometimes people can get competitive and wrapped up in it." People use puzzles to show off their smarts, make others feel subordinate, and enjoy telling the story of the game later (as I'm doing right now).

Of course, the fervor with which some people debate basic arithmetic may be a proxy: There's less at stake in a math debate than a potentially friendship-ending political debate. Arguing over multiplication may even be a way to make a subtle political point, using others' "wrong" answers to reinforce a broader worldview, such as that the United States has poor math education.

But why do the debates often go on so long? One reason is psychological, another mathematical.

Math is already a source of anxiety for many people, and adding an audience ups the ante. "When there's an audience, your performance can change," says Sian Beilock of the University of Chicago and author of Choke: What the Secrets of the Brain Reveal About Getting It Right When You Have To. Emotional stress can overtake our limited reserves of "cognitive horsepower." "Often people feel the most stressed when the audience is made up of people they know. It's very painful to fall on your face in front of your friends and family."

So people dig in, not realizing the other reason these debates drag on—a mathematical one. We are taught to think of math as an absolute discipline without ambiguity. To an extent, that's true: Two plus two is always four. But while the math itself lacks ambiguity, the way we express that math requires a system of symbols—otherwise known as language. Consider how often people debate grammar. Math has syntax just as language does—with the same potential for ambiguities. And just as word-based riddles exploit the ambiguities of language, so do these math problems.

Some of you are already insisting in your head that 6 ÷ 2(1+2) has only one right answer, but hear me out. The problem isn't the mathematical operations. It's knowing what operations the author of the problem wants you to do, and in what order. Simple, right? We use an "order of operations" rule we memorized in childhood: "Please excuse my dear Aunt Sally," or PEMDAS, which stands for Parentheses Exponents Multiplication Division Addition Subtraction.* This handy acronym should settle any debate—except it doesn't, because it's not a rule at all. It's a convention, a customary way of doing things we've developed only recently, and like other customs, it has evolved over time. (And even math teachers argue over order of operations.)

"In earlier times, the conventions didn't seem as rigid and people were supposed to just figure it out if they were mathematically competent," says Judy Grabiner, a historian of mathematics at Pitzer College in Claremont, Calif. Mathematicians generally began their written work with a list of the conventions they were using, but the rise of mass math education and the textbook industry, as well as the subsequent development of computer programming languages, required something more codified. That codification occurred somewhere around the turn of the last century. The first reference to PEMDAS is hard to pin down. Even a short list of what different early algebra texts taught reveals how inconsistently the order of operations was applied.

So that brings us back to 6 ÷ 2(1+2). There are three ways to think about this problem—and none is incorrect. (If you don't believe me, plug it into a few different calculators, or even check out Google, where commenters have argued over Google's calculator answer.)

One way is to interpret the obelus, or ÷ symbol, as dividing everything to the left of it by everything to the right of it. Textbooks don't typically use the symbol that way today, but it has been used that way historically. If you calculate the problem using this convention, it's 6 divided by (2(1+2)), which is 1. Typically, though, if the author wanted you to interpret it that way, she would have used parentheses to indicate as much.

You can alternatively apply PEMDAS as schools do today: Simplify everything inside the parentheses first, then exponents, then all multiplication and division from left to right in the order both operations appear, then all addition and subtraction from left to right in the order both operations appear. (A better acronym would be PEMA, actually, to make it clear that multiplication and division are done together, and addition and subtraction are done together.) By that convention, $6 \div 2(1+2) = 6 \div 2 \times (1+2) = 6 \div 2 \times 3 = 3 \times 3 = 9$. If you were taking the ACT, SAT, or GRE (which would probably use parentheses to eliminate confusion), this method would yield the correct answer.

But wait, you say—isn't that 2 to the left of the parentheses part of simplifying the parentheses? After all, this is what my own Facebook debate partners were arguing. In fact, the 2 is not part of the "P" in PEMDAS for simplifying parentheses, but there is a basis for simplifying the 2(2+1) before it's divided by 6. It's called "implied multiplication by juxtaposition." We know the expression 5x means to multiply 5 and x because they are juxtaposed next to one other. But should these operations be done before a division that occurs to the left of them in a problem? That depends on whom you're talking to, or what calculator or programming language you're using.

Internet rumors claim the American Mathematical Society has written "multiplication indicated by juxtaposition is carried out before division," but no original AMS source exists online anymore (if it ever did). Still, some early math textbooks also taught students to do all multiplications and then all divisions, but most, such as this 1907 high-school algebra textbook, this 1910 textbook, and this 1912 textbook, recommended performing all multiplications and divisions in the order they appear first, followed by additions and subtractions. (This convention makes sense as well with the Canadian and British versions of PEMDAS, such as BEDMAS, BIDMAS, and BODMAS, which all list division before multiplication in the acronym.) The most sensible advice, in a 1917 edition of Mathematical Gazette, recommended using parentheses to avoid ambiguity. (Duh!) But even noted math historian Florian Cajori wrote in A History of Mathematical Notations in 1928-29, "If an arithmetical or algebraical term contains ÷ and ×, there is at present no agreement as to which sign shall be used first."

If it “feels” natural to you that implied multiplication takes precedence over division (whether because it’s next to a parentheses or not), then you would get $6 \div 2(1+2) = 6 \div (2(3)) = 6 \div 6 = 1$. That answer would be incorrect on most U.S. standardized tests, but you wouldn’t necessarily be wrong. (Insert rant against standardized tests here.) You would just be in the minority about which convention you’re using.

Still unconvinced that arguing over math problems is similar to arguing over whether to use a plural or singular pronoun with indefinite pronouns? Let’s return to the obelus (÷) because a brief history of division signs reveals the ambiguity of the syntax of math. Nearly a half-dozen division signs have been recorded in mathematical notation. The colon was used in a 1633 text, which seems odd until you realize we still use it in ratios (2:3 is commonly the same as 2/3 in ratios).

Even before that, a close parentheses was used in the 1540s, so that 8)24 meant $24 \div 8$. Again, that looks odd, but we still use it today in long division. It just looks different because we combine it with a different symbol, the lengthy vinculum (--------) across the top, to group together the numbers to be divided. The vinculum is also used over repeating decimal digits and with radicals (is used with -------- across the top); you probably just didn’t realize the square root sign was a mashup of two math symbols. A vinculum usually has little to do with division; it’s used in fractions and to group together numbers just as parentheses are.

You might expect $10 \div 5$ is the same as 10/5 is the same as 10 over a 5 with a vinculum between them, but each has its own eccentricities. We’ve already noted that ÷ can mean “divide the number on the left by the number on the right” or “divide the expression on the left by the expression on the right.” But it gets really tricky when people assume that a slash replaces a vinculum. Does ab/cd = (ab)÷(cd) or ((ab)÷c)÷d? Does a/b/c mean (a)÷(b)÷(c) or a÷(b/c) or (a/b)÷c? (Answer: Use some parentheses!)

The bottom line is that “order of operations” conventions are not universal truths in the same way that the sum of 2 and 2 is always 4. Conventions evolve throughout history in response to cultural and technological shifts. Meanwhile, those ranting online about gaps in U.S. math education and about the “right” answer to these intentionally ambiguous math problems might be, ironically, missing a bigger point.

“To my mind,” says Grabiner, “the major deficit in U.S. math education is that people think math is about calculation and formulas and getting the one right answer, rather than being about exciting ideas that cut across all sorts of intellectual categories, clear and logical thinking, the power of abstraction and a language that lets you solve problems you’ve never seen before.” Even if that language, like any other, can be a bit ambiguous sometimes.

Correction, March 12, 2013: This article originally misstated the mnemonic for the PEMDAS order of operations rule. It's "excuse," not "forgive." (Return to the corrected sentence.)

Mathematics & Intuition: Sometimes Strange Bedfellows

June 07, 2012 | Author: Jeff Zilahy |

Our intuition and our ability to discern what is going on around us is a key component of being human. We really need to be able to calculate the chances of events in our given environment. This is a fundamental skill that has prevented us from being eaten by lions (at least most of us!) and in turn effectively propagate our species over time. So in a sense, one of the integral ingredients of humanity is being adept at constantly analyzing questions related to probability.

Generally speaking, probability is the branch of mathematics that deals with chance. Probability and science are inextricably linked, as developments in our understanding of probability helped to write the laws of science over the last three centuries. This in turn greatly impacted fields like biology, physics, and psychology. In the business world, people who work in probability are often known as actuaries, where they analyze risk, thus forming the bedrock of the insurance industry. The chance of thunderstorms, the chance of getting a perfect score on a test, the chance of surviving any situation, much of our lives stem from our ability to have a sense of the odds of any given situation. Therefore, when we encounter situations in which the probability appears obvious but is not readily apparent, it can be a bit jarring and even cause disbelief, at least until a careful explanation is made. One elementary example is called the "Two children problem". The problem has several variations but is generally stated as the following.

Two children problem

Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys? Take a moment and think about your answer. Did you say 1/2? If you did, you are most certainly...not quite correct! No, in fact, the generally accepted answer is the seemingly bizarre 1/3. Before exclaiming this result as incorrect, consider the answer will likely make more sense when we examine the possible scenarios that can occur for Mr. Smith's children. He could have a boy, then a girl, or he could have a girl, then a boy, or a boy and then a boy or a girl and then a girl. These are the four possible outcomes for Mr. Smith, or anyone else that has two children. Now, given our original question, we know that with our available options (e.g. BB, BG, GB and GG), the girl and girl scenario can be ruled out since we know there is at least one boy. That leaves three possible scenarios for Mr. Smith: boy then a boy, or boy then a girl, or girl then a boy. We want to know the probability that they are both boys, and since there is only one such scenario, we now know that there is a one out of three chance that they are both boys. Are you still surprised? Now, granted, there is some controversy surrounding this problem as it pertains to the subjective nature inherent in the interpretation of the question. However, the important point to make here is the realization that our intuition does not always guide us to the correct answer. Now, I would be remiss if I didn't leave you with a final quandary to ponder. As a sort of exercise left for the reader, try to figure out the answer to the following variation to the Two Children problem. I have two children, one of whom is a son born on a Tuesday. What is the probability that I have two boys? Did you say 13/27? That is what I thought!

Lesson 22 Summary: April 16th
Descartes & the Cartesian Plane
Slope, Distance, Midpoint Formulas

Lesson Handouts
Descartes' La Geometrie (Cliff Pickover, The Math Book)
Points in the Coordinate plane worksheet (Kuta)
Parallel Lines in the Coordinate Plane worksheet (Kuta)
Graphing Lines worksheet (Kuta)
The Distance Formula worksheet (Kuta)
The Midpoint Formula worksheet (Kuta)
Descartes Rules of Signs worksheet (Kuta)

Biologists think they are biochemists,
Biochemists think they are Physical Chemists,
Physical Chemists think they are Physicists,
Physicists think they are Gods,
And God thinks he is a Mathematician.
-Plato

Mar 12-1:33 PM

WARM UP:

1. Use of change of base formula to calculate $\log_5(1/625)$.

2. What is $\log_x(1)$ = ?

3. What is a cipher? Provide an original cipher system.

4. If there is a cipher where you move up 3 letters in the alphabet, translate the following, "WHFKQRORJB"

5. **Age and Loans** Suppose 20% of the population are 65 or over, 26% of those 65 or over have loans, and 53% of those under 65 have loans. Find the probabilities that a person fits into the following categories.
   a. 65 or over and has a loan
   b. Has a loan

Feb 6-9:21 AM

WARM UP:

$\frac{\ln(1/625)}{\ln 5}$

1. Use of change of base formula to calculate $\log_5(1/625)$.

2. What is $\log_x(1)$ = ? $x^{?} = 1$

3. What is a cipher? Provide an original cipher system.

4. If there is a cipher where you move up 3 letters in the alphabet, translate the following, "WHFKQRORJB"

5. **Age and Loans** Suppose 20% of the population are 65 or over, 26% of those 65 or over have loans, and 53% of those under 65 have loans. Find the probabilities that a person fits into the following categories.
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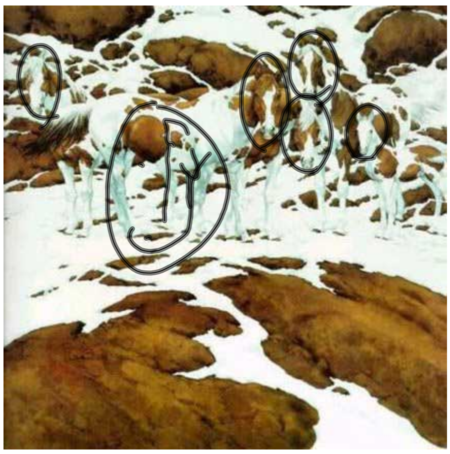
Feb 6-9:21 AM

Visual Fun for the Day:



How many horses?

Feb 11-12:17 PM

History of Mathematics
RESEARCH PROJECT

This will represent 20% of your final grade. (per syllabus)
It will be due 4/25/13.
It needs to be at least 3 pages typed (double spaced, font 12)
The paper should be original thoughts only, no copy and pasting!
You choose one of the following options:

PAST
Write a report that carefully traces the ***technology of mathematics*** from the earliest days of civilization through the 20th century. You should be sure to cover how developments like the abacus and slide rule enhanced and improved society.

PRESENT
Read the ebook, **"A Cultural Paradox: Fun in Mathematics"**, then write a response report that goes into detail on at least five of the topics covered.

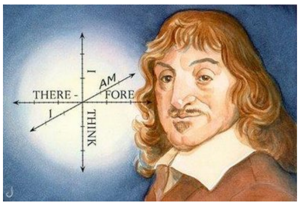
FUTURE
Conduct in-depth research on the resource **Wolfram Alpha**.What is its history? What can it be used for? How is it different/better than Google? Be sure you include at least 5 original search queries with accompanying results in your report.

Apr 11-11:33 AM

"Cogito ergo sum"

aka

"I think, therefore I am."



These famous words were first uttered by the French philosopher and mathematician Rene Descartes. (31 March 1596 – 11 February 1650)

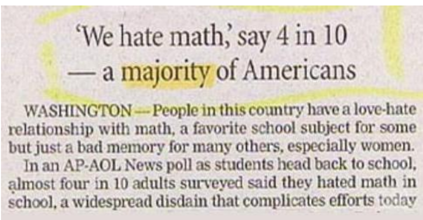
Descartes' had a tremendous influence in mathematics, the Cartesian coordinate system is named after him. This allowed a point in space to be represented with numbers, AND allowing algebraic equations to be expressed as geometric shapes in a two-dimensional coordinate system, and therefore, shapes to be described as equations!

Apr 15-10:27 AM

ALGEBRA MEETS GEOMETRY

The genius of Descartes is he developed a way to bridge the *abstract nature of Algebra* with the *visual nature of Geometry*. He is credited as the father of analytical geometry. His work helped pave the way for the development of Calculus, without which we could kiss most of the modern world, and all its lovely conveniences, goodbye.

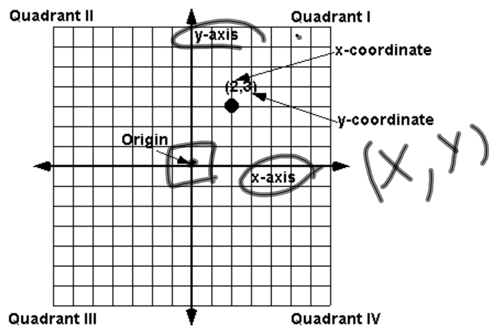
Yet.......


'We hate math,' say 4 in 10 — a majority of Americans

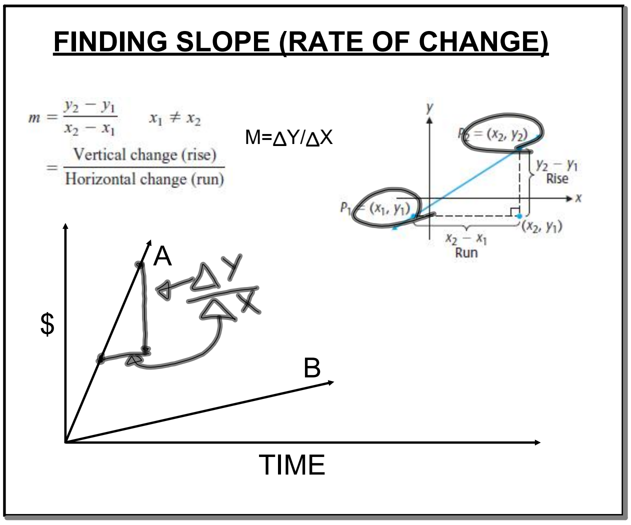
WASHINGTON — People in this country have a love-hate relationship with math, a favorite school subject for some but just a bad memory for many others, especially women.
In an AP-AOL News poll as students head back to school, almost four in 10 adults surveyed said they hated math in school, a widespread disdain that complicates efforts today

Apr 16-10:40 AM

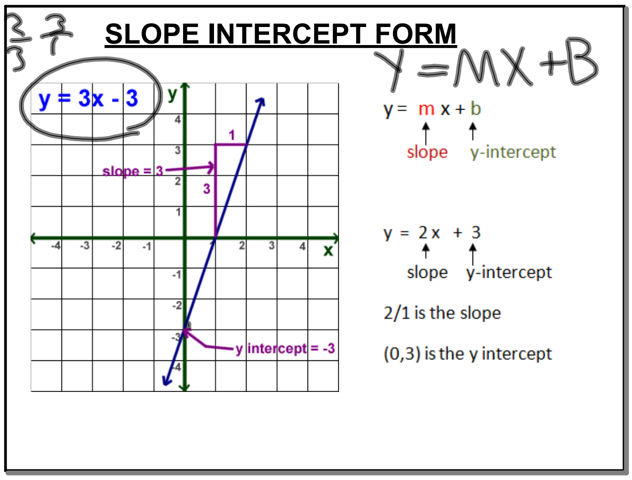
OVERVIEW OF THE CARTESIAN PLANE



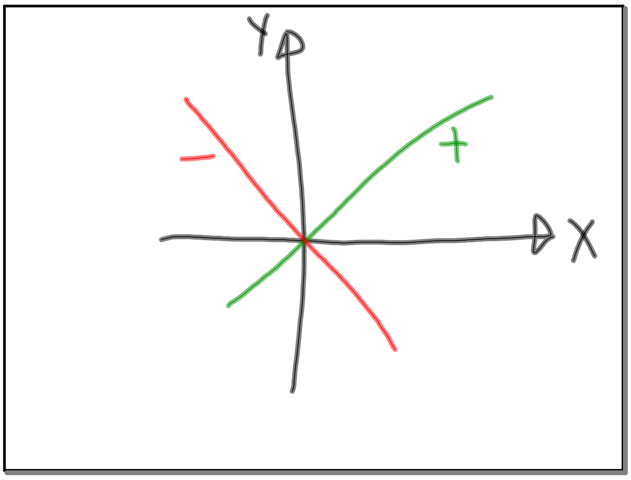
Apr 16-11:17 AM



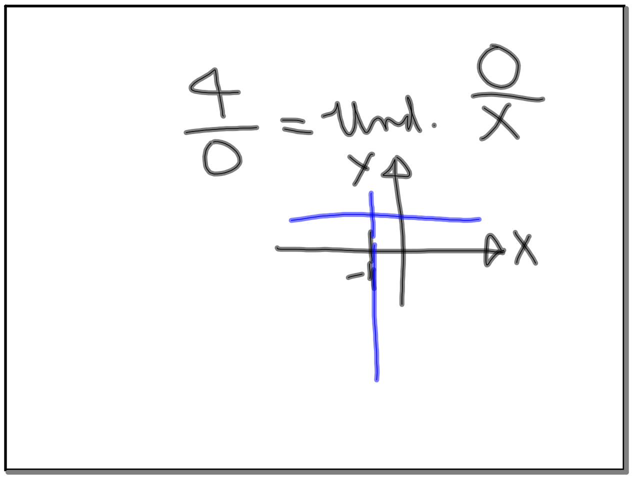
Apr 16-11:19 AM



Apr 11-11:23 AM



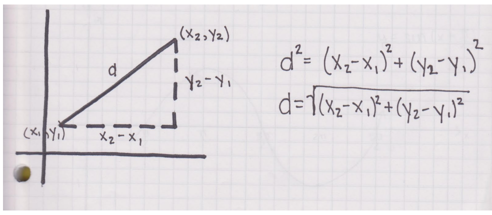
Apr 16-2:00 PM



Apr 16-1:49 PM

DISTANCE FORMULA

The Distance Formula is a variant of the Pythagorean Theorem



Given the two points (x1, y1) and (x2, y2), the distance between these points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Apr 16-11:38 AM

Mid-point Formula

$$\left(\frac{x_1 + x_2}{2} \, , \, \frac{y_1 + y_2}{2} \right)$$

Apr 16-11:49 AM

Descartes' Rule of Signs

This is a useful technique for finding the zeroes of a polynomial.

MAX=4
2 or 0

Translation: we are able to find where the function equals zero, in many cases, this is very useful information related to the nature of the function.

f(x) = x^5 - x^4 + 3x^3 + 9x^2 - x + 5

f(-x) ✓ ✓ ✓ ✓

Apr 16-11:51 AM

Apr 16-12:41 PM

Descartes' *La Géométrie*

René Descartes (1596–1650)

In 1637, French philosopher and mathematician René Descartes published *La géométrie*, which shows how geometrical shapes and figures can be analyzed using algebra. Descartes' work influenced the evolution of analytical geometry, a field of mathematics that involves the representation of positions in a coordinate system and in which mathematicians algebraically analyze such positions. *La géométrie* also shows how to solve mathematical problems and discusses the representation of points of a plane through the use of real numbers, and the representation and classification of curves through the use of equations.

Interestingly, *La géométrie* does not actually use "Cartesian" coordinate axes or any other coordinate system. The book pays as much attention to representing algebra in geometric forms as vice versa. Descartes believed that algebraic steps in a proof should usually correspond to a geometrical representation.

Jan Gullberg writes, "*La géométrie* is the earliest mathematical text that a modern student of mathematics could read without stumbling over an abundance of obsolete notations....Along with Newton's *Principia*, it is one of the most influential scientific texts of the seventeenth century." According to Carl Boyer, Descartes desired to "free geometry" from the use of diagrams through algebraic procedures and to give meaning to the operations of algebra through geometric interpretation.

More generally, Descartes was groundbreaking in his proposal to unite algebra and geometry into a single subject. Judith Grabiner writes, "Just as the history of Western philosophy has been viewed as a series of footnotes to Plato, so the past 350 years of mathematics can be viewed as a series of footnotes to Descartes' *Geometry*...and the triumph of Descartes' methods of problem solving."

Boyer concludes, "In terms of mathematical ability, Descartes probably was the most able thinker of his day, but he was at heart not really a mathematician." His geometry was only one facet of a full life that revolved around science, philosophy, and religion.

SEE ALSO Pythagorean Theorem and Triangles (c. 600 B.C.), Quadrature of the Lune (c. 440 B.C.), Euclid's *Elements* (300 B.C.), Pappus's Hexagon Theorem (c. 340), Projective Geometry (1639), and Fractals (1975).

The Ancient of Days *(1794), a watercolor etching by William Blake. European medieval scholars often associated geometry and the laws of nature with the divine. Through the centuries, geometry's focus on compass and straightedge constructions became more abstract and analytical.*

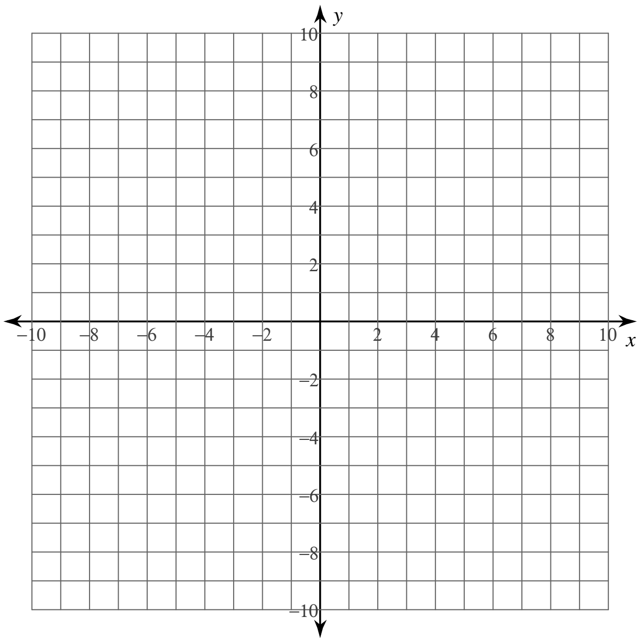
Kuta Software - Infinite Geometry   Name___________________________________

Points in the Coordinate Plane

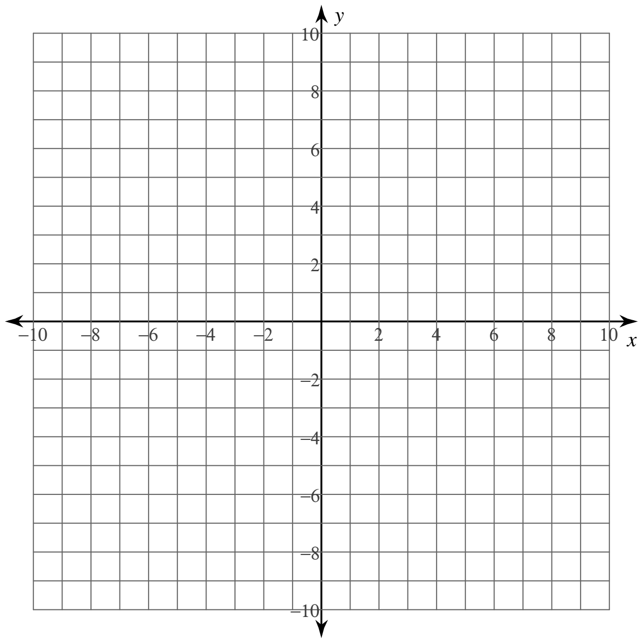
Date________________ Period____

Plot each point.

1) $J(5, 10)$ $I(1, 9)$ $H(6, -9)$
$G(-6, 8)$ $F(9, 0)$ $E(-6, 0)$
$D(-8, -4)$ $C(5, 0)$ $B(-1, -1)$
$A(-8, -1)$

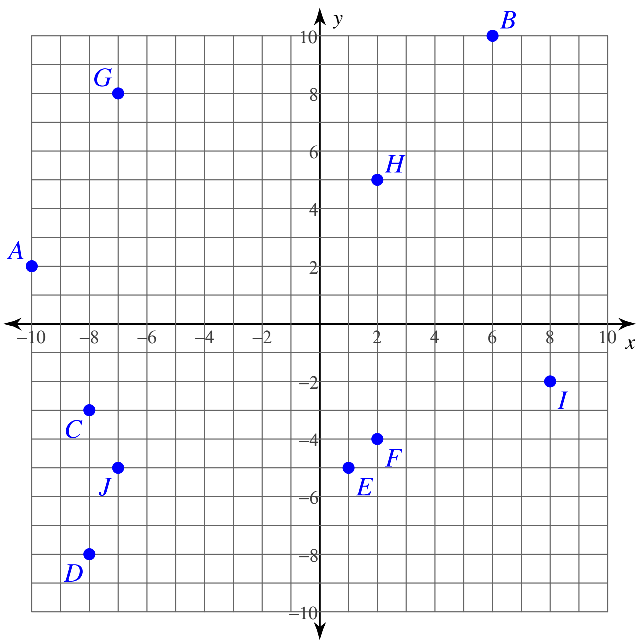


2) $A(7, 10)$ $B(0, 4)$ $C(-1, 10)$
$D(-6, -6)$ $E(10, 0)$ $F(9, 7)$
$G(-3, -4)$ $H(-4, -9)$ $I(4, 1)$
$J(7, -9)$

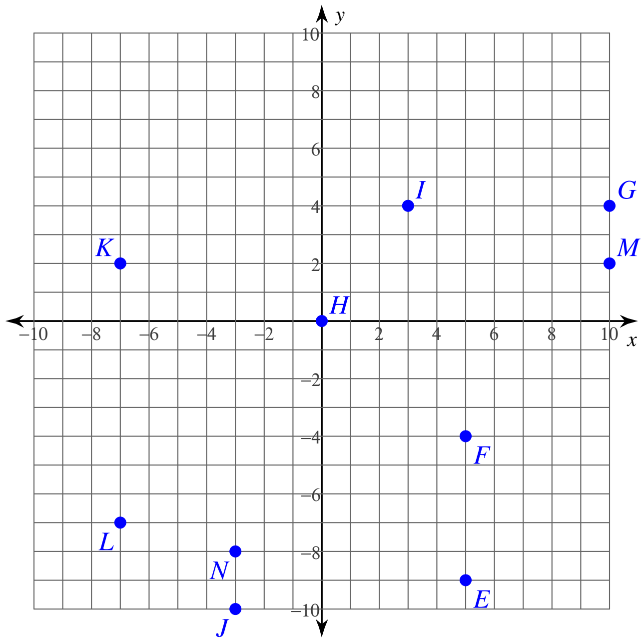


State the coordinates of each point.

3)



4)



State the quadrant or axis that each point lies in.

5) $L(-2, 1)$ $K(-3, -2)$ $J(3, 1)$

6) $T(-3, 5)$ $U(1, 0)$ $V(-5, 5)$

7) $S(5, -7)$ $T(7, 2)$ $U(-5, 4)$

8) $R(7, 0)$ $Q(8, -1)$ $P(3, 0)$

Critical thinking questions:

9) State the coordinates of the endpoints of a line segment that intersects the y-axis.

10) State the coordinates of the endpoints of a line segment that is not parallel to either axis, and does not intersect either axis.

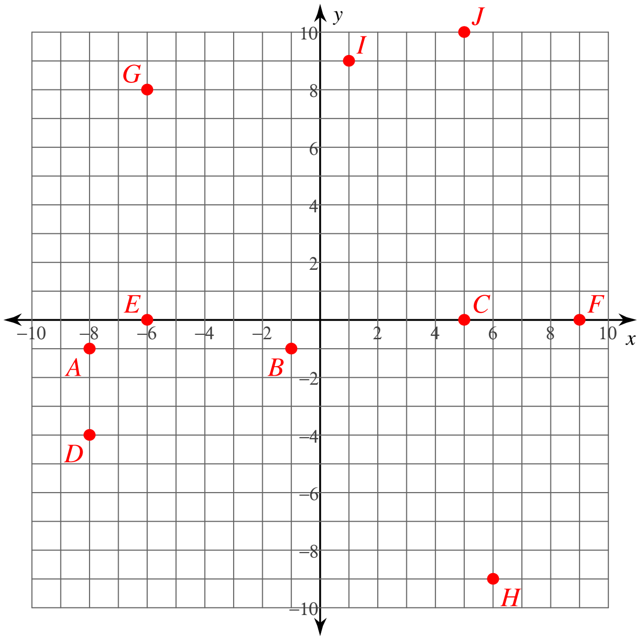
Name________________________________

Points in the Coordinate Plane

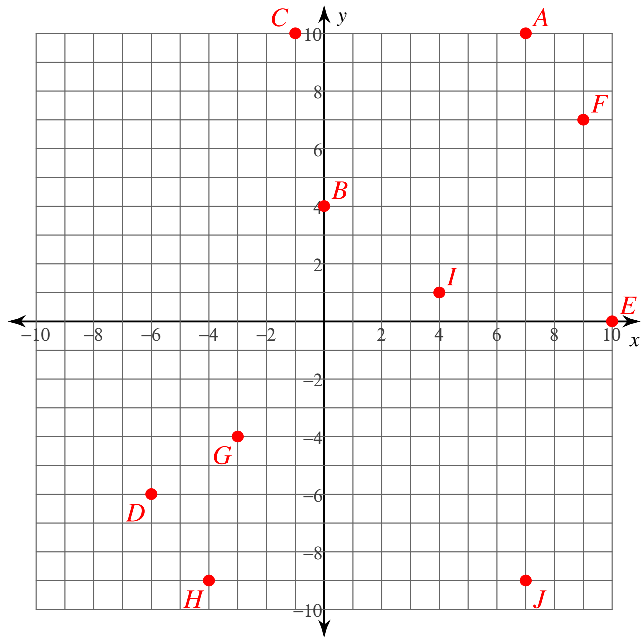
Date________________ Period____

Plot each point.

1) $J(5, 10)$ $I(1, 9)$ $H(6, -9)$
$G(-6, 8)$ $F(9, 0)$ $E(-6, 0)$
$D(-8, -4)$ $C(5, 0)$ $B(-1, -1)$
$A(-8, -1)$

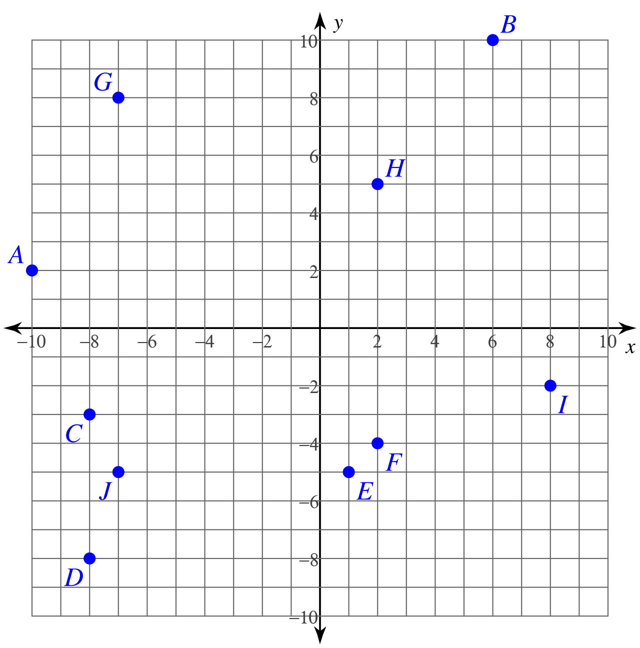


2) $A(7, 10)$ $B(0, 4)$ $C(-1, 10)$
$D(-6, -6)$ $E(10, 0)$ $F(9, 7)$
$G(-3, -4)$ $H(-4, -9)$ $I(4, 1)$
$J(7, -9)$



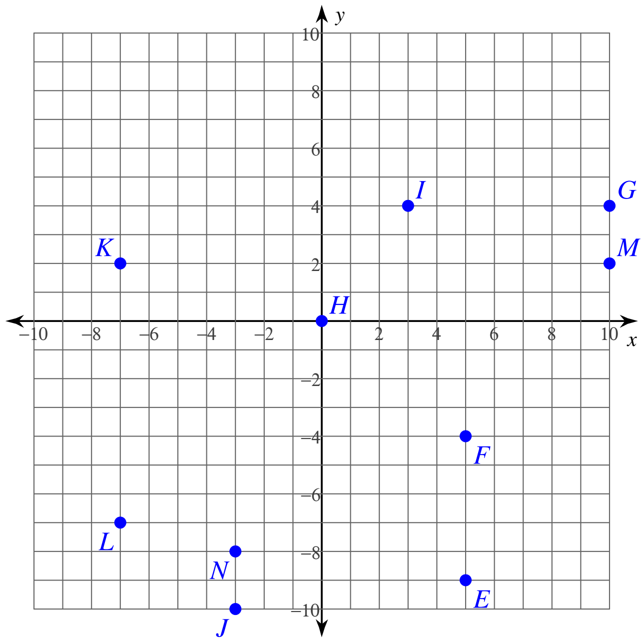
State the coordinates of each point.

3)



$A(-10, 2)$ $B(6, 10)$ $C(-8, -3)$
$D(-8, -8)$ $E(1, -5)$ $F(2, -4)$
$G(-7, 8)$ $H(2, 5)$ $I(8, -2)$
$J(-7, -5)$

4)



$E(5, -9)$ $F(5, -4)$ $G(10, 4)$
$H(0, 0)$ $I(3, 4)$ $J(-3, -10)$
$K(-7, 2)$ $L(-7, -7)$ $M(10, 2)$
$N(-3, -8)$

State the quadrant or axis that each point lies in.

5) $L(-2, 1)$ $K(-3, -2)$ $J(3, 1)$

L: II K: III J: I

6) $T(-3, 5)$ $U(1, 0)$ $V(-5, 5)$

T: II U: x-axis V: II

7) $S(5, -7)$ $T(7, 2)$ $U(-5, 4)$

S: IV T: I U: II

8) $R(7, 0)$ $Q(8, -1)$ $P(3, 0)$

R: x-axis Q: IV P: x-axis

Critical thinking questions:

9) State the coordinates of the endpoints of a line segment that intersects the y-axis.

Many answers. Ex: $(2, 2)$, $(-2, 2)$

10) State the coordinates of the endpoints of a line segment that is not parallel to either axis, and does not intersect either axis.

Many answers. Ex: $(2, 2)$, $(3, 3)$

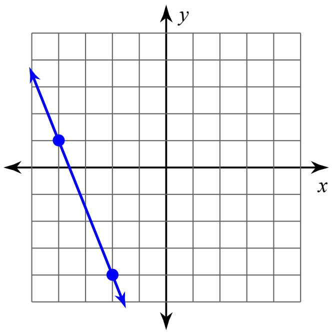
Name________________________________

Parallel Lines in the Coordinate Plane

Date________________ Period____

Find the slope of each line.

1)



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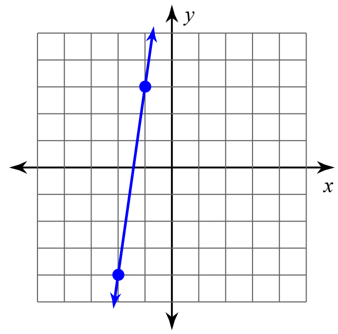
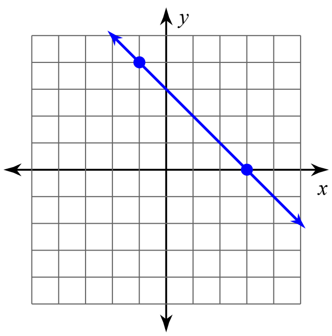
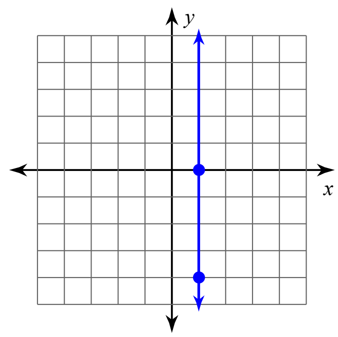


3)



4)



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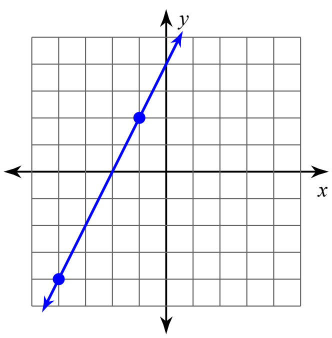
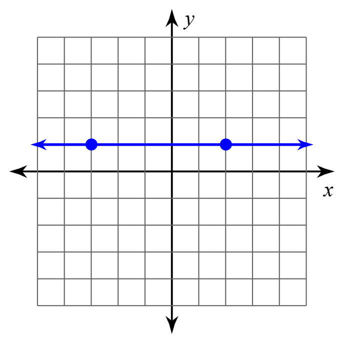


6)



7) $y = -\frac{1}{3}x - 4$

8) $y = 2x - 2$

9) $x = -1$

10) $y = \frac{3}{2}x - 3$

11) $y = -\frac{7}{5}x - 3$

12) $y = -\frac{5}{4}x - 2$

Write the slope-intercept form of the equation of each line given the slope and y-intercept.

13) Slope = −3, y-intercept = −1

14) Slope = $\frac{5}{3}$, y-intercept = −3

15) Slope = −1, y-intercept = 3

16) Slope = $\frac{2}{5}$, y-intercept = 1

17) Slope = 3, y-intercept = 0

18) Slope = $-\frac{1}{2}$, y-intercept = 4

Find the slope of a line parallel to each given line.

19) $y = 2x - 5$

20) $y = 2x - 4$

21) $y = \frac{4}{5}x - 3$

22) $y = -\frac{8}{3}x - 4$

23) $y = -x - 2$

24) $y = -2x - 1$

Critical thinking questions:

25) Fill in the blank so that the lines are not parallel:

Line A goes through	Line B goes through
$(0, 8)$ and $(-2, 0)$	$(1, 2)$ and $(3, \_\_)$

26) Write the equations of five lines that are parallel to $y = \frac{x}{2} - 6$

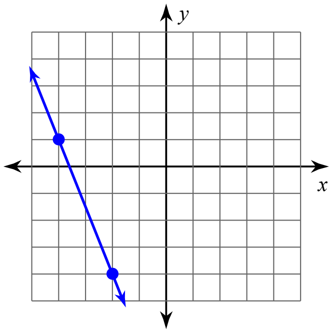
Kuta Software - Infinite Geometry Name________________________________

Parallel Lines in the Coordinate Plane

Date________________ Period____

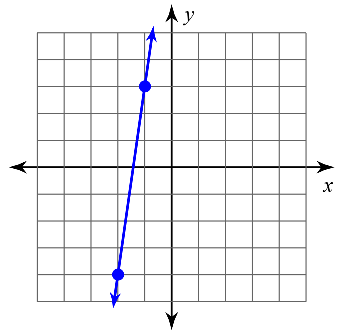
Find the slope of each line.

1)



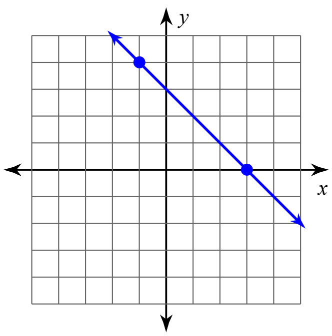
$-\frac{5}{2}$

2)



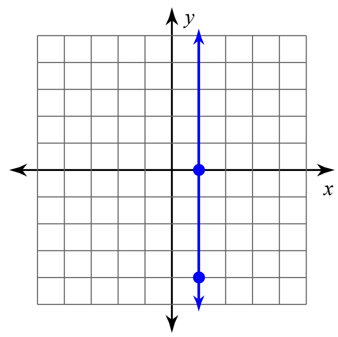
7

3)



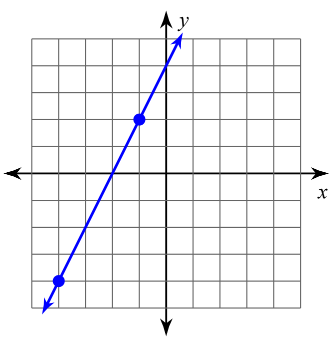
-1

4)



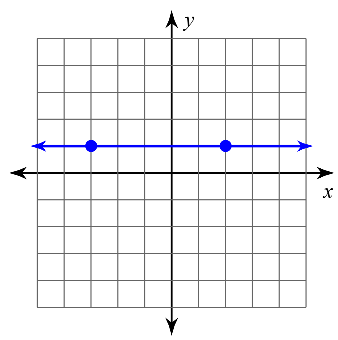
Undefined

5)



2

6)



0

7) $y = -\frac{1}{3}x - 4$

$-\frac{1}{3}$

8) $y = 2x - 2$

2

9) $x = -1$

Undefined

10) $y = \frac{3}{2}x - 3$

$\frac{3}{2}$

11) $y = -\frac{7}{5}x - 3$

$-\frac{7}{5}$

12) $y = -\frac{5}{4}x - 2$

$-\frac{5}{4}$

Write the slope-intercept form of the equation of each line given the slope and y-intercept.

13) Slope $= -3$, y-intercept $= -1$

$y = -3x - 1$

14) Slope $= \frac{5}{3}$, y-intercept $= -3$

$y = \frac{5}{3}x - 3$

15) Slope $= -1$, y-intercept $= 3$

$y = -x + 3$

16) Slope $= \frac{2}{5}$, y-intercept $= 1$

$y = \frac{2}{5}x + 1$

17) Slope $= 3$, y-intercept $= 0$

$y = 3x$

18) Slope $= -\frac{1}{2}$, y-intercept $= 4$

$y = -\frac{1}{2}x + 4$

Find the slope of a line parallel to each given line.

19) $y = 2x - 5$

2

20) $y = 2x - 4$

2

21) $y = \frac{4}{5}x - 3$

$\frac{4}{5}$

22) $y = -\frac{8}{3}x - 4$

$-\frac{8}{3}$

23) $y = -x - 2$

-1

24) $y = -2x - 1$

-2

Critical thinking questions:

25) Fill in the blank so that the lines are not parallel:

Line A goes through	Line B goes through
$(0, 8)$ and $(-2, 0)$	$(1, 2)$ and $(3, \_\_)$

Anything but 10

26) Write the equations of five lines that are parallel to $y = \frac{x}{2} - 6$

Many answers. Ex: $y = \frac{x}{2} + 4$

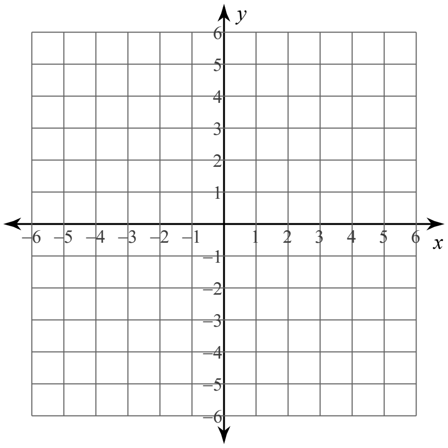
Name____

Graphing Lines

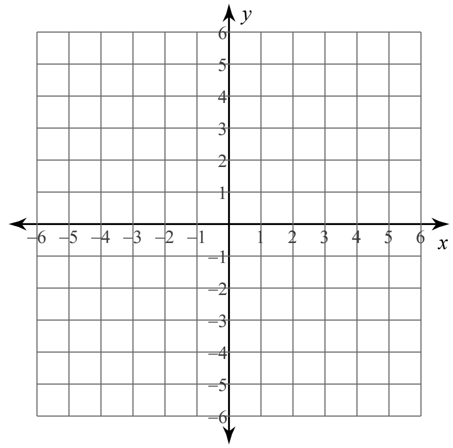
Date________________ Period____

Sketch the graph of each line.

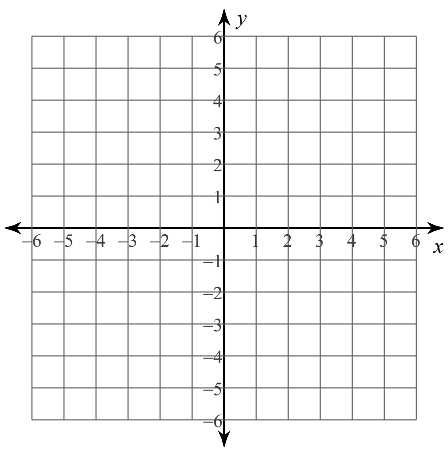
1) $y = \frac{7}{2}x - 2$



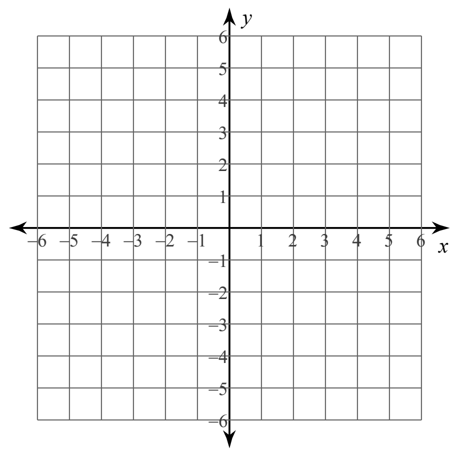
2) $y = -6x + 3$



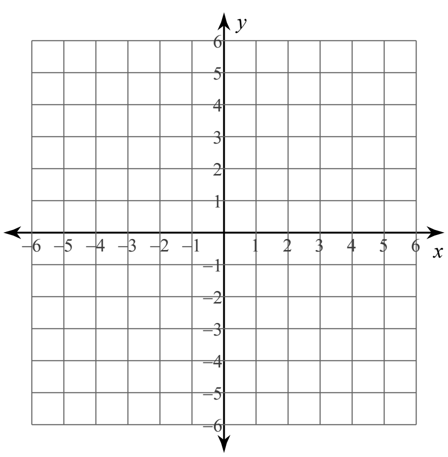
3) $y = -5$



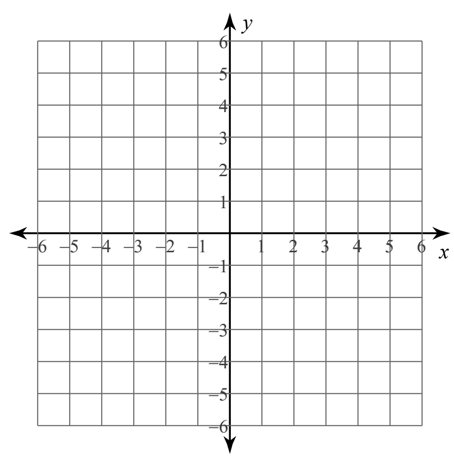
4) $y = \frac{6}{5}x + 1$



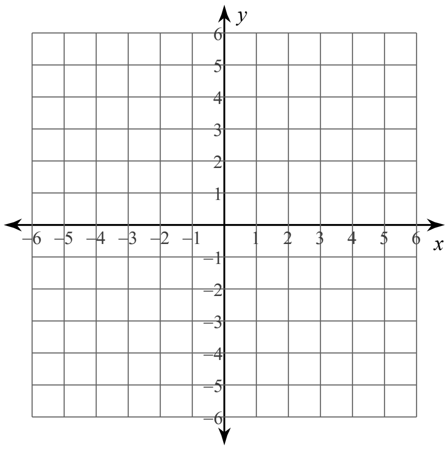
5) $y = \frac{1}{4}x + 2$



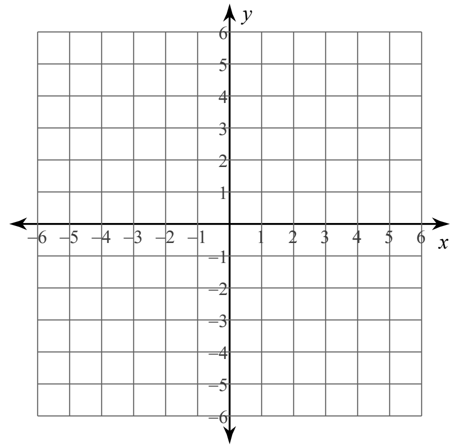
6) $x = 5$



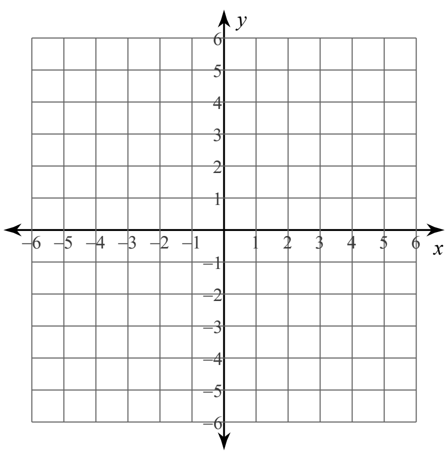
7) $y = \frac{5}{3}x$



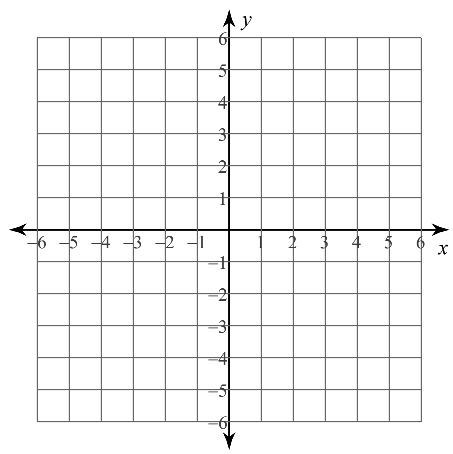
8) $x = 0$



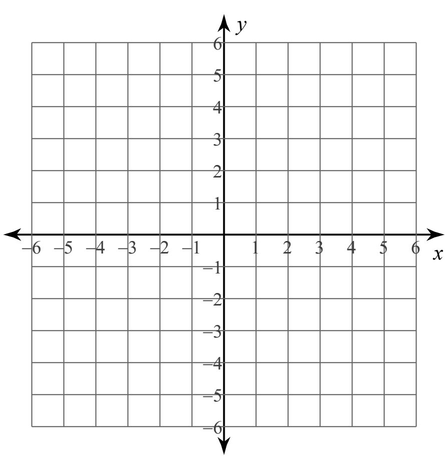
9) $y = -\frac{1}{3}x + 3$



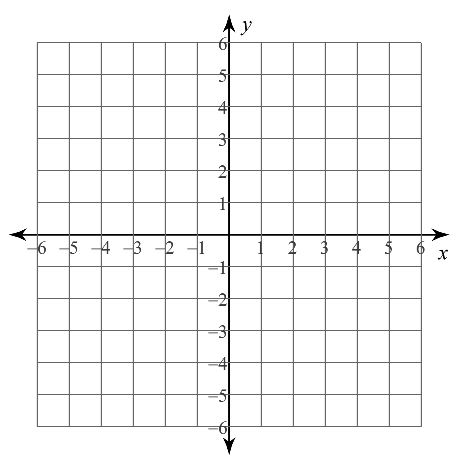
10) $y = \frac{1}{5}x - 4$



11) $y = \frac{1}{2}x - 2$



12) $y = 2x + 5$



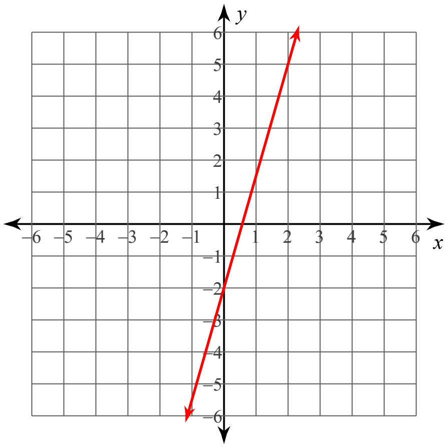
Name________________________________

Graphing Lines

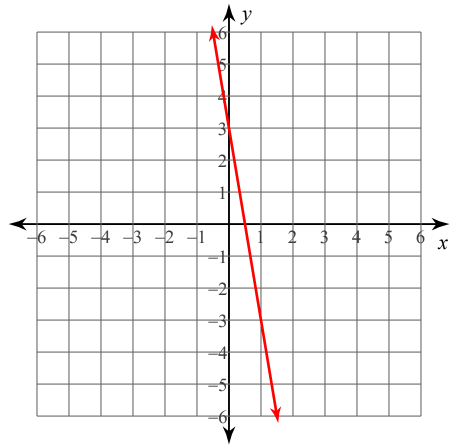
Date________________ Period____

Sketch the graph of each line.

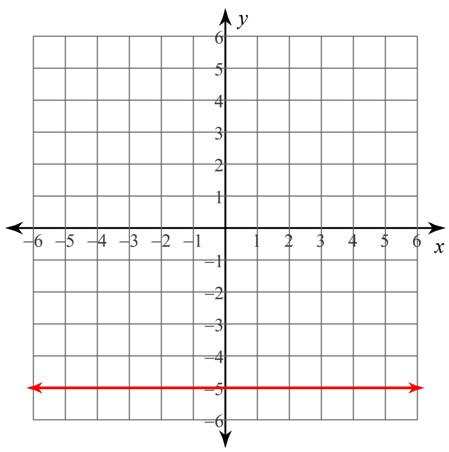
1) $y = \frac{7}{2}x - 2$



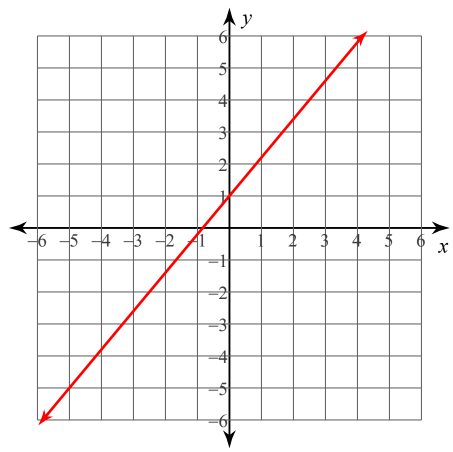
2) $y = -6x + 3$



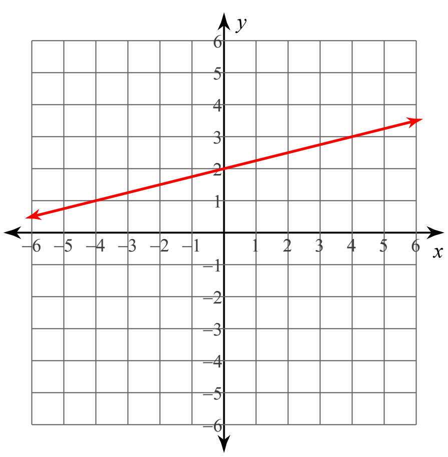
3) $y = -5$



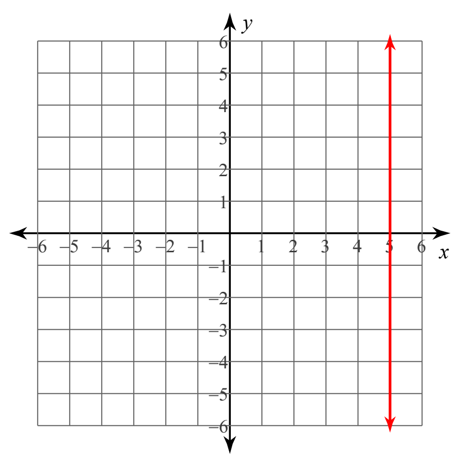
4) $y = \frac{6}{5}x + 1$



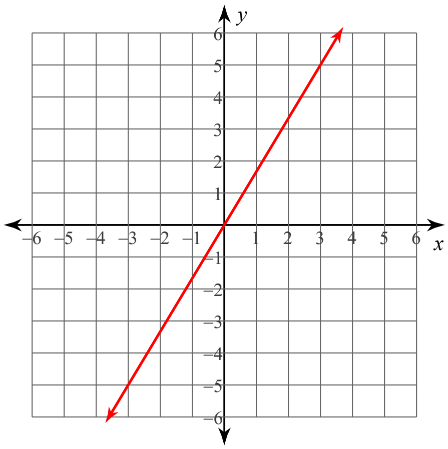
5) $y = \frac{1}{4}x + 2$



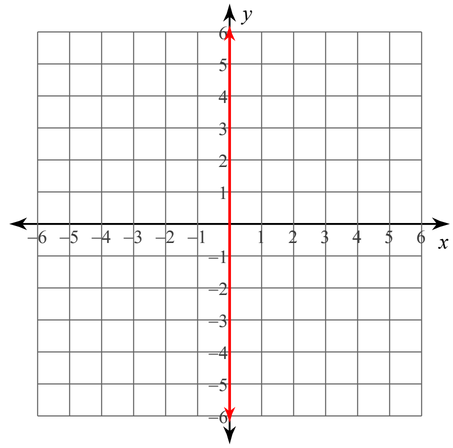
6) $x = 5$



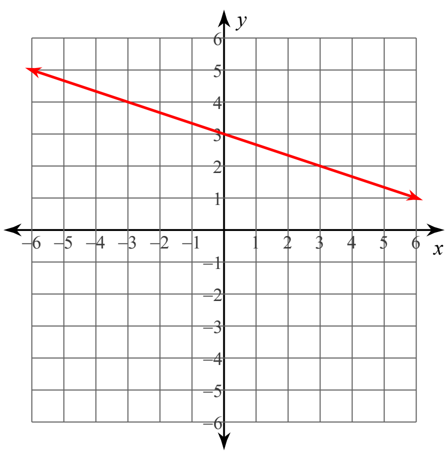
7) $y = \frac{5}{3}x$



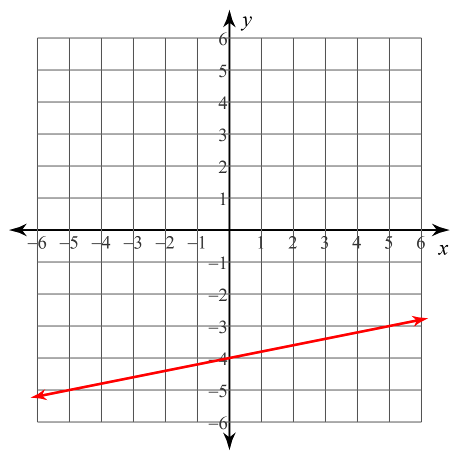
8) $x = 0$



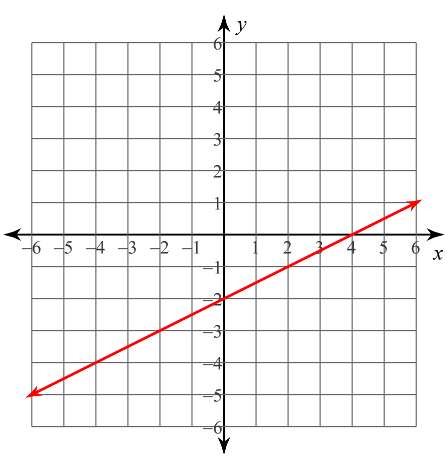
9) $y = -\frac{1}{3}x + 3$



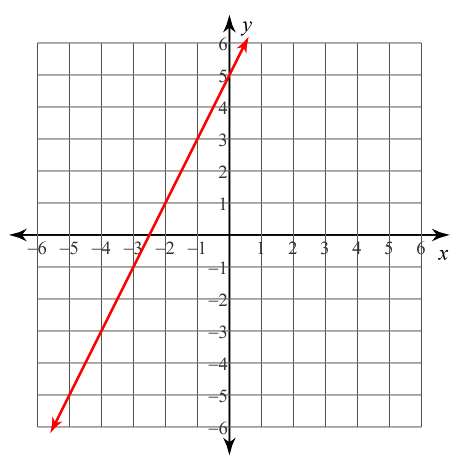
10) $y = \frac{1}{5}x - 4$



11) $y = \frac{1}{2}x - 2$



12) $y = 2x + 5$



Kuta Software - Infinite Pre-Algebra

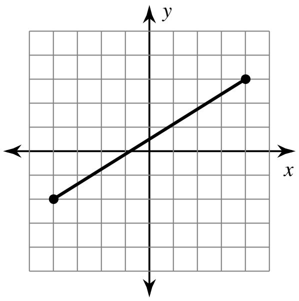
Name___________________________________

The Distance Formula

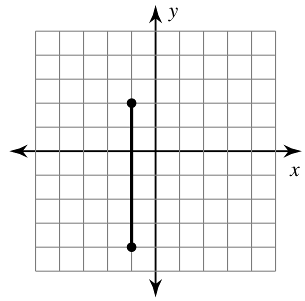
Date________________ Period____

Find the distance between each pair of points.

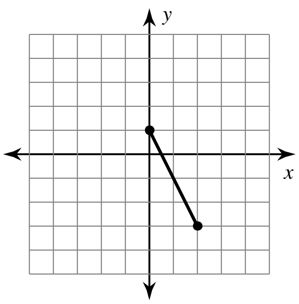
1)



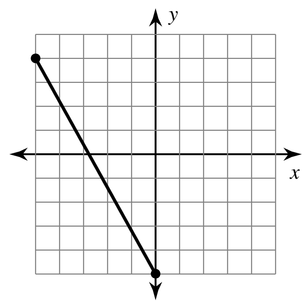
2)



3)



4)



5) $(-1, 2), (2, -4)$

6) $(4, 3), (-3, 4)$

7) $(0, 4), (2, 3)$

8) $(4, 0), (-4, 1)$

9) $(12, 12), (-3, 1)$

10) $(1, -9), (6, -6)$

11) $(5, -10), (-5, 4)$

12) $(5, 5), (-6, -4)$

Kuta Software - Infinite Pre-Algebra

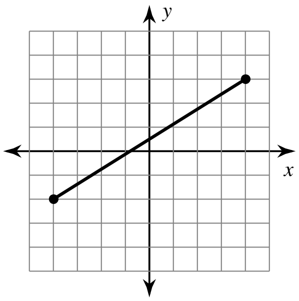
Name________________________________

The Distance Formula

Date________________ Period____

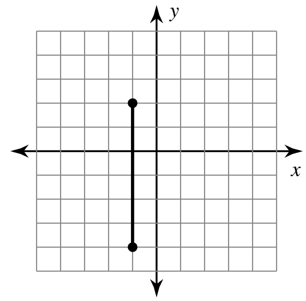
Find the distance between each pair of points.

1)



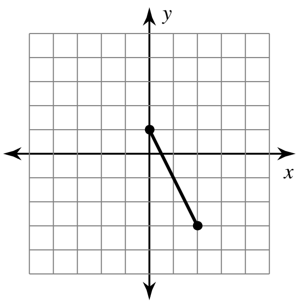
9.433

2)



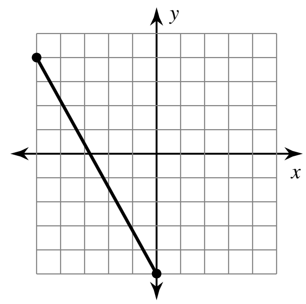
6

3)



4.472

4)



10.295

5) $(-1, 2),\ (2, -4)$

6.708

6) $(4, 3),\ (-3, 4)$

7.071

7) $(0, 4),\ (2, 3)$

2.236

8) $(4, 0),\ (-4, 1)$

8.062

9) $(12, 12),\ (-3, 1)$

18.601

10) $(1, -9),\ (6, -6)$

5.83

11) $(5, -10),\ (-5, 4)$

17.204

12) $(5, 5),\ (-6, -4)$

14.212

Kuta Software - Infinite Pre-Algebra

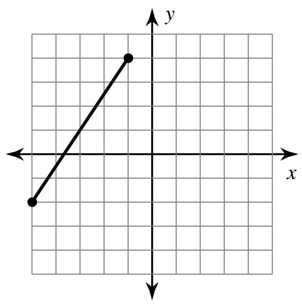
Name____________________

The Midpoint Formula

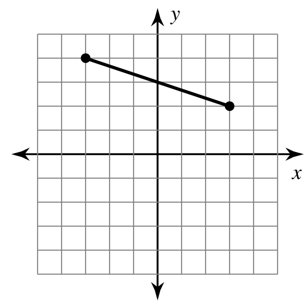
Date________________ Period____

Find the midpoint of each line segment.

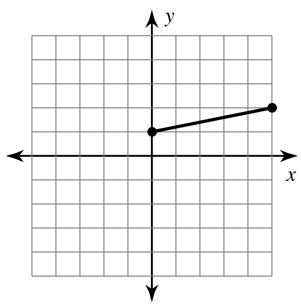
1)



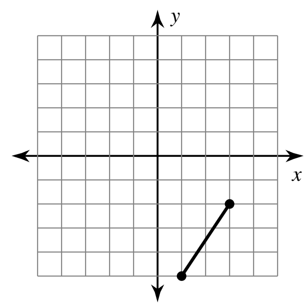
2)



3)



4)



Find the midpoint of the line segment with the given endpoints.

5) $(6, 1)$, $(-2, 5)$

6) $(4, 3)$, $(-1, 6)$

7) $(-5, 4)$, $(-6, 3)$

8) $(4, -3)$, $(5, 5)$

9) $(11, -10)$, $(-3, 9)$

10) $(8, 2)$, $(9, 8)$

11) $(5, 1)$, $(-4, -12)$

12) $(-8, -9)$, $(7, 7)$

13) $(-9, 8)$, $(0, -11)$

14) $(10, 9)$, $(0, -10)$

Kuta Software - Infinite Pre-Algebra

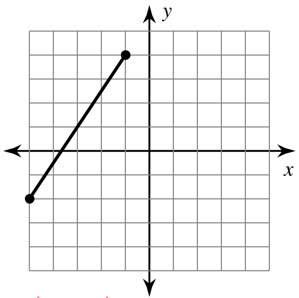
Name________________________

The Midpoint Formula

Date________________ Period____

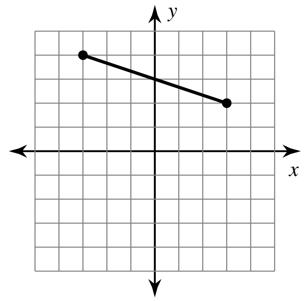
Find the midpoint of each line segment.

1)



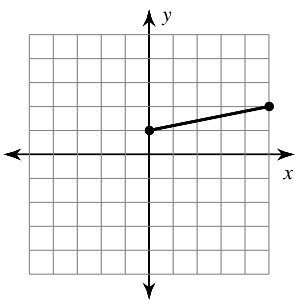
$(-3, 1)$

2)

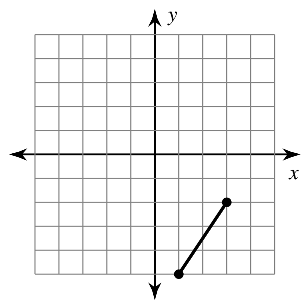


$(0, 3)$

3) $\left(2\frac{1}{2}, 1\frac{1}{2}\right)$



4) $\left(2, -3\frac{1}{2}\right)$



Find the midpoint of the line segment with the given endpoints.

5) $(6, 1)$, $(-2, 5)$

$(2, 3)$

6) $(4, 3)$, $(-1, 6)$

$(1.5, 4.5)$

7) $(-5, 4)$, $(-6, 3)$

$(-5.5, 3.5)$

8) $(4, -3)$, $(5, 5)$

$(4.5, 1)$

9) $(11, -10)$, $(-3, 9)$

$(4, -0.5)$

10) $(8, 2)$, $(9, 8)$

$(8.5, 5)$

11) $(5, 1)$, $(-4, -12)$

$(0.5, -5.5)$

12) $(-8, -9)$, $(7, 7)$

$(-0.5, -1)$

13) $(-9, 8)$, $(0, -11)$

$(-4.5, -1.5)$

14) $(10, 9)$, $(0, -10)$

$(5, -0.5)$

Kuta Software - Infinite Algebra 2 Name________________________________

Descartes' Rule of Signs

Date________________ Period____

State the possible number of positive and negative zeros for each function.

1) $f(x) = 3x^4 + 20x^2 - 32$

2) $f(x) = 5x^4 - 42x^2 + 49$

3) $f(x) = 4x^3 - 12x^2 - 5x + 1$

4) $f(x) = 2x^4 - 3x^3 + x$

5) $f(x) = 2x^4 + 3x^2 - 54$

6) $f(x) = x^6 - 64$

7) $f(x) = 9x^6 - 3x^5 + 33x^4 - 11x^3 + 18x^2 - 6x$

8) $f(x) = 64x^6 - 1$

9) $f(x) = 2x^5 + 4x^4 + 9x^3 + 18x^2 - 35x - 70$

10) $f(x) = 6x^5 - 4x^4 - 63x^3 + 42x^2 + 147x - 98$

11) $f(x) = 16x^6 - 32x^4 - 25x^2 + 50$

12) $f(x) = x^7 - 64x$

13) $f(x) = x^6 - 64$

14) $f(x) = 8x^6 + 9x^3 + 1$

15) $f(x) = 27x^6 + 26x^3 - 1$

16) $f(x) = 27x^9 - x^6 - 27x^3 + 1$

17) $f(x) = 16x^8 - 73x^4 + 36$

18) $f(x) = 9x^8 - 106x^4 + 225$

19) $f(x) = x^6 - 64$

20) $f(x) = 16x^8 - 153x^4 + 81$

Critical thinking questions:

21) Write a polynomial function that has 0 possible positive real zeros and 5, 3, or 1 possible negative real zero.

 Name____________________

Descartes' Rule of Signs

Date________________ Period____

State the possible number of positive and negative zeros for each function.

1) $f(x) = 3x^4 + 20x^2 - 32$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

2) $f(x) = 5x^4 - 42x^2 + 49$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 2 or 0

3) $f(x) = 4x^3 - 12x^2 - 5x + 1$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 1

4) $f(x) = 2x^4 - 3x^3 + x$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 1

5) $f(x) = 2x^4 + 3x^2 - 54$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

6) $f(x) = x^6 - 64$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

7) $f(x) = 9x^6 - 3x^5 + 33x^4 - 11x^3 + 18x^2 - 6x$

Possible # positive real zeros: 5, 3, or 1
Possible # negative real zeros: 0

8) $f(x) = 64x^6 - 1$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

9) $f(x) = 2x^5 + 4x^4 + 9x^3 + 18x^2 - 35x - 70$

Possible # positive real zeros: 1
Possible # negative real zeros: 4, 2, or 0

10) $f(x) = 6x^5 - 4x^4 - 63x^3 + 42x^2 + 147x - 98$

Possible # positive real zeros: 3 or 1
Possible # negative real zeros: 2 or 0

11) $f(x) = 16x^6 - 32x^4 - 25x^2 + 50$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 2 or 0

12) $f(x) = x^7 - 64x$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

13) $f(x) = x^6 - 64$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

14) $f(x) = 8x^6 + 9x^3 + 1$

Possible # positive real zeros: 0
Possible # negative real zeros: 2 or 0

15) $f(x) = 27x^6 + 26x^3 - 1$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

16) $f(x) = 27x^9 - x^6 - 27x^3 + 1$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 1

17) $f(x) = 16x^8 - 73x^4 + 36$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 2 or 0

18) $f(x) = 9x^8 - 106x^4 + 225$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 2 or 0

19) $f(x) = x^6 - 64$

Possible # positive real zeros: 1
Possible # negative real zeros: 1

20) $f(x) = 16x^8 - 153x^4 + 81$

Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 2 or 0

Critical thinking questions:

21) Write a polynomial function that has 0 possible positive real zeros and 5, 3, or 1 possible negative real zero.

Many answers. Ex. $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$

Lesson 23 Summary: April 18th

Probability

Rope Around the Earth Puzzle

Pascal's Triangle

Combinations

Lesson Handouts

N/A

"Small minds discuss persons.

Average minds discuss events.

Great minds discuss ideas.

Really great minds discuss mathematics."

~Anon

WARM UP:

1. Why is the Cartesian plane so powerful?

2. List as many ways, including formulas, to express slope.

$\frac{RISE}{RUN}$, $\frac{y_2 - y_1}{x_2 - x_1}$, M,

3. What is the slope and y intercept in y = −7x + 1/2?

WARM UP:

1. Why is the Cartesian plane so powerful?
It connects algebra and geometry.

2. List as many ways, including formulas, to express slope.
Slope, m, rate of change, Δy/Δx, y2-y1/x2-x1, rise/run, change in y/change in x, vertical change/horizontal change...

3. What is the slope and y intercept in y = -7x + 1/2?
slope=-7/1 and y-int = 1/2

73. Cigarette Smokers The following table gives a recent estimate (in millions) of the smoking status among persons 25 years of age and over and their highest level of education. *Source: National Health Interview Survey.*

Education	Current Smoker	Former Smoker	Non-Smoker	Total
Less than a high school diploma	7.90	6.66	14.12	28.68
High school diploma or GED	14.38	13.09	25.70	53.17
Some college	12.41	13.55	28.65	54.61
Bachelor's degree or higher	4.97	12.87	38.34	56.18
Total	39.66	46.17	106.81	192.64

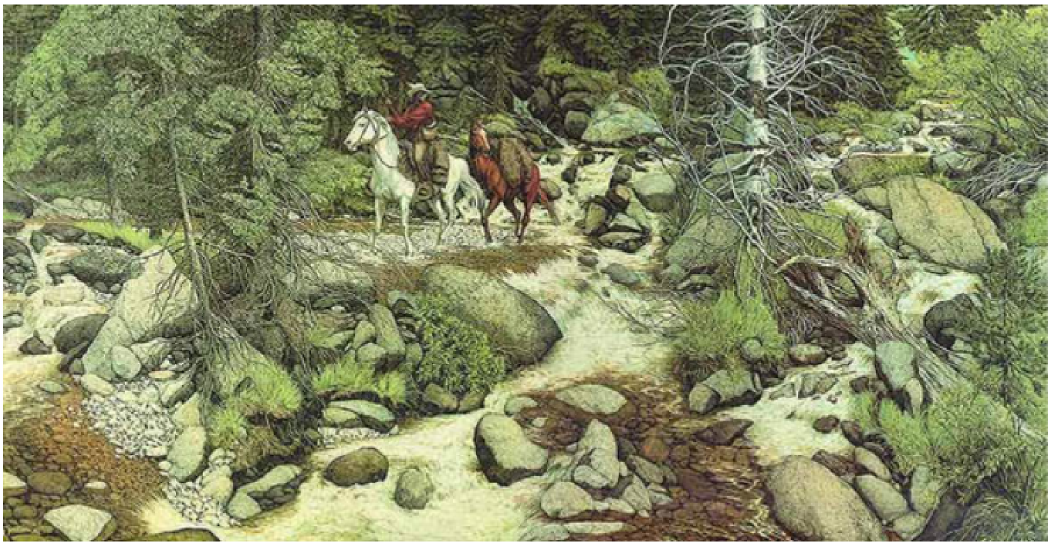
a. Find the probability that a person is a current smoker.

b. Find the probability that a person has less than a high school diploma.

c. Find the probability that a person is a current smoker and has less than a high school diploma.

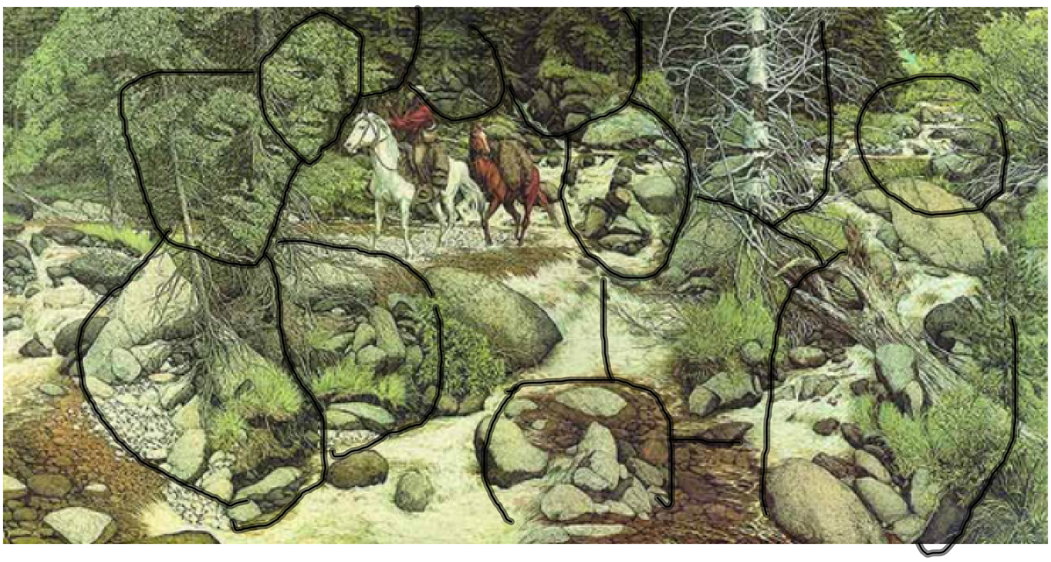
d. Find the probability that a person is a current smoker, given that the person has less than a high school diploma.

Visual Fun for the Day:



How many faces?

Visual Fun for the Day:



How many faces? ANS: 13

Let's review the answers to the worksheets from Tuesday.

Given the two points (x1, y1) and (x2, y2), the distance between these points is given by th...

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2 P+1 = −2, 0

1 P+2 = 3, −5

−2, −5

3, 0

Mid-point Formula

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{-4+5}{2}, \frac{-1+3}{2}\right)$$

(0.5, 1)

Descartes' Rule of Signs

This is a useful technique for finding the zeroes of a polynomial.

MAX=4
2 or 0

Translation: we are able to find where the function equals zero, in many cases, this is very useful information related to the nature of the function.

f(x) = x^5 - x^4 + 3x^3 + 9x^2 - x + 5

f(-x)

ROPE AROUND THE EARTH PUZZLE (1702)

Imagine a rope tightly encircling around a basketball. How much longer would the rope have to be to be 1 foot from the surface of the basketball at all points? Now think about a rope tightly encircling the Earth at its widest point (equator). This would be a rope about 25,000 miles long.

QUESTION: How much **extra rope** would you need for it to be one foot from the surface of earth at all points?

HINT: What is the formula for the circumference of a sphere? **C= 2πr**

HINT2: What is the value for our new r in this problem? **1 + r**

ANS: The answer is the same for both the basketball and earth, the shockingly small: 2π, or approximately 6.28 feet for both. Consider, if r is the radius of the Earth (or basketball), then 1 + r is the radius in feet of the enlarged circle. Therefore, we can compare the rope circumference before (2πr) and after (2π(1 + r)). The difference is 2π (when you distribute)

Pascal's Triangle (1654)

Pascal's Triangle is a triangle shaped array of numbers, named after Blaise Pascal, a famous French Mathematician and Philosopher.

There are many patterns that emerge from Pascal's triangle; geometric patterns, integer patterns, perfect square patterns. We can even find the Fibonacci sequence and we can create fractal patterns if we replace the numbers by dots and gaps.

Let's now look at how to construct one!

BUILDING PASCAL'S

1. To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.

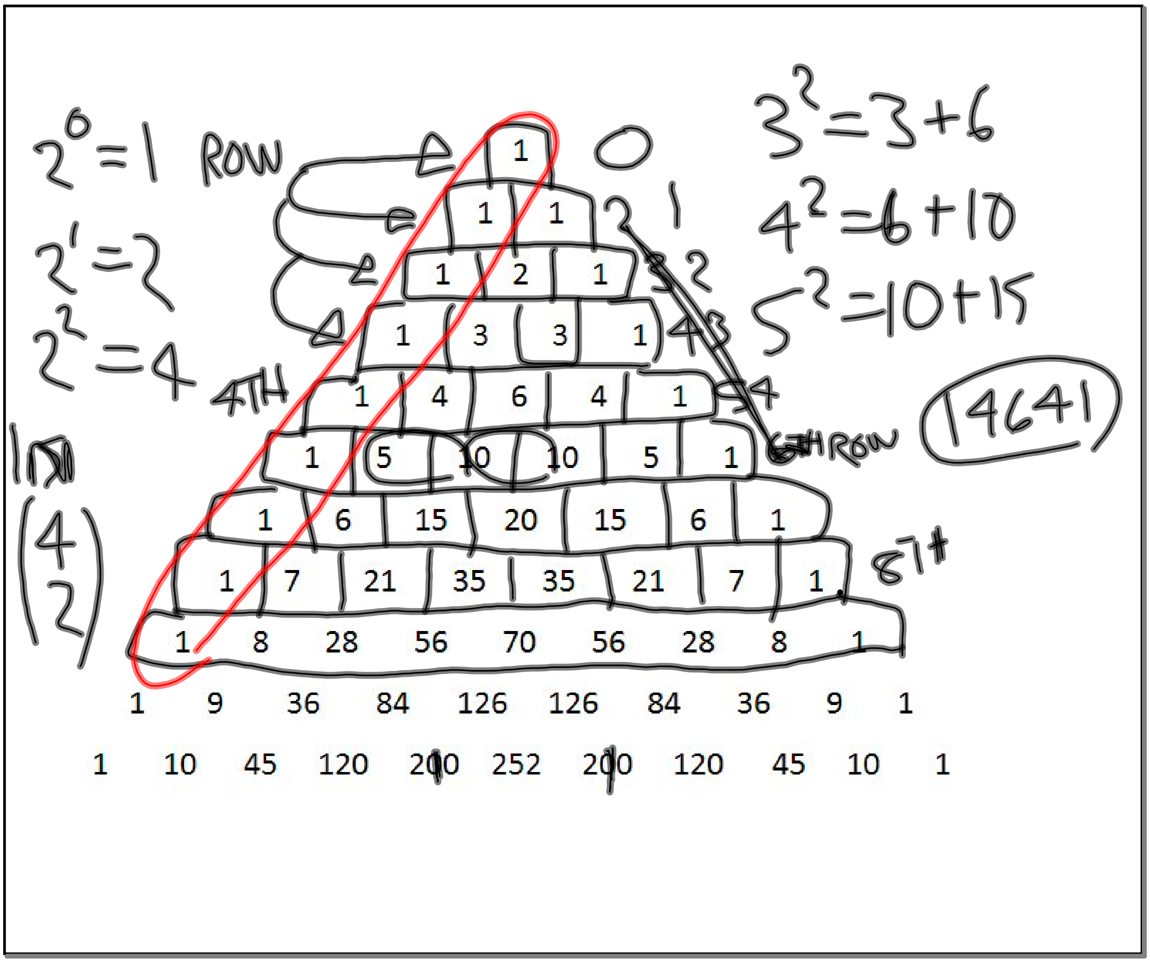
2. Each number is just the two numbers above it added together (except for the edges, which are all "1").

3. You start out with the top two rows: 1, and 1 1.

4. Then to construct each entry in the next row, you look at the two entries above it (i.e. the one above it and to the right, and the one above it and to the left). At the beginning and the end of each row, when there's only one number above, put a 1.

5. You might even think of this rule (for placing the 1's) as included in the first rule: for instance, to get the first 1 in any line, you add up the number above and to the left (since there is no number there, pretend it's zero) and the number above and to the right (1), and get a sum of 1.

6. Write at least 8 rows of the triangle.



PASCAL'S TRIANGLE QUESTIONS:

1. What numbers make up the first diagonal?

2. What numbers make up the second diagonal?

3. If you consider a number in the second diagonal, and compare it with the sum of the number to the right and below it, what relationship emerges?

4. What do you notice about the horizontal sums?

5. If you consider each row as a single number then row = $x^{\text{row number}}$. What is x?

6. If you draw a line down the center of the triangle, what can you say about the two sides?

One idea in math is how many ways we can choose distinct items from a larger group.

We call the number of ways to select "k" objects from "n" total objects is called a **combination**.

$7! = 7 \cdot 6 5 4 \cdot 3 \cdot 2 \cdot 1$

The formula for calculating a combination is given by:

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

TRY: In how many ways can we make a group of 3 students out of the 10 total students in this course?

ANS: 10 C 3 = 120 ways to make a group of 3 students.

Well, another cool feature of Pascal's Triangle is it allows us to find the answer to any combination problem like above.

HOW?
We consider each row to correspond to our N value and our K value to correspond to our place in the row. NOTE: The first row=0 and the first place in any row=0.

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

TRY: What would the 6 in Pascal's Triangle represent if expressed as a combination? Use the combination notation!

QUESTION:
There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference. How many different ways are there to select a group of four students to attend the conference?

$$\binom{37}{4} \qquad \frac{37!}{4!\,33!} = \frac{37 \cdot \overset{3}{\cancel{36}} \cdot 35 \cdot \overset{17}{\cancel{34}}}{4!\,\cancel{4} \cdot \cancel{3} \cdot \cancel{2}}$$

ANS: 14 juniors, 23 seniors, 37 students total

$_{37}C_4 = 66{,}045$

QUESTION:
Eleven students put their names on slips of paper inside a box. Three names are going to be taken out. How many different ways can the three names be chosen?

ANS: 11c3 = 165

Lesson 24 Summary: April 23rd
Calculus: Newton & Leibniz
Derivative as Slope
Difference Quotient

Lesson Handouts
Discovery of Calculus (Cliff Pickover, The Math Book)
Difference Quotient Examples Handout

"To most outsiders, modern mathematics is unknown territory.
Its borders are protected by dense thickets of technical terms; its landscapes are a mass of indecipherable equations and incomprehensible concepts.
Few realize that the world of modern mathematics is rich with vivid images and provocative ideas."

~Ivars Peterson

WARM UP:

1. In Pascal's Triangle, if a row represents a number, what is the number in row 0? Row 9?

2. If you have three distinct objects, how many ways can we pick 2 objects? 1 object? 3 objects?

3. Using your calculator, what is 12c5?

4. **Women Joggers** In a certain area, 15% of the population are joggers and 40% of the joggers are women. If 55% of those who do not jog are women, find the probabilities that an individual from that community fits the following descriptions.

   a. A woman jogger

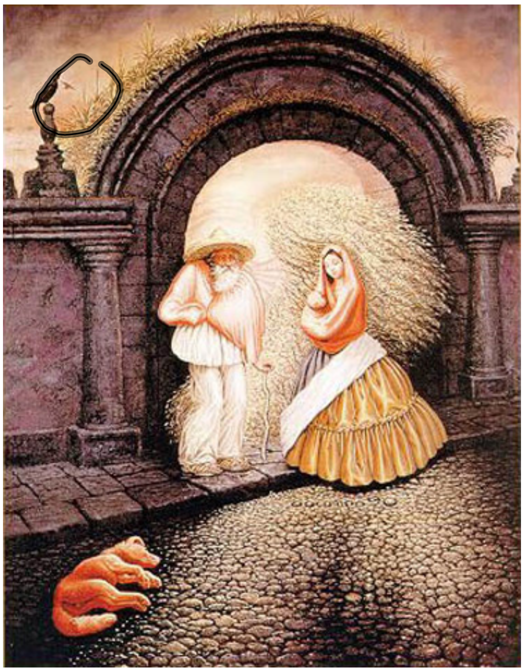
   b. A man who is not a jogger

   c. A woman

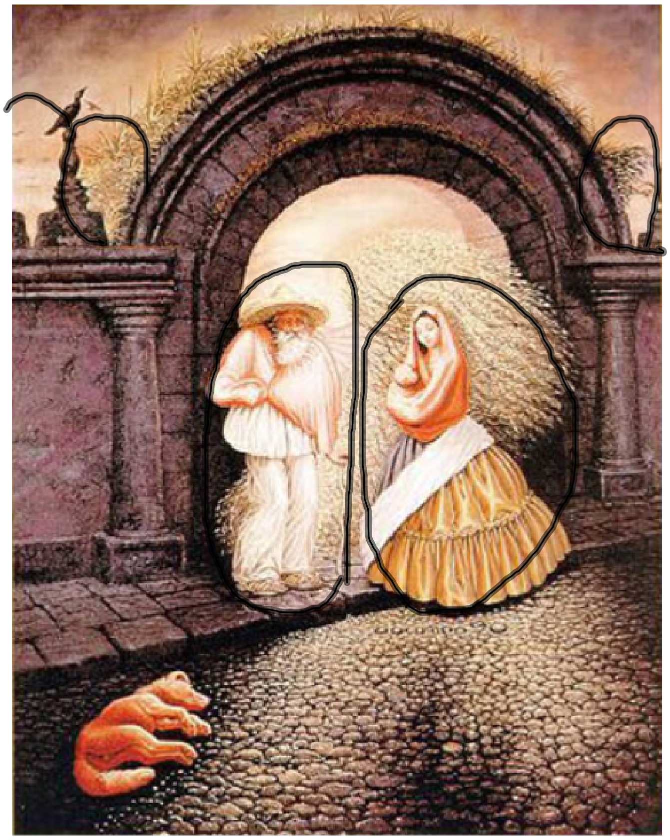
Visual Fun for the Day:

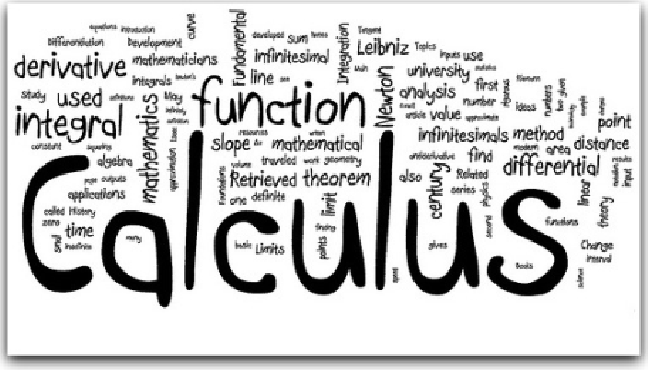
How many F's does the following passage contain?

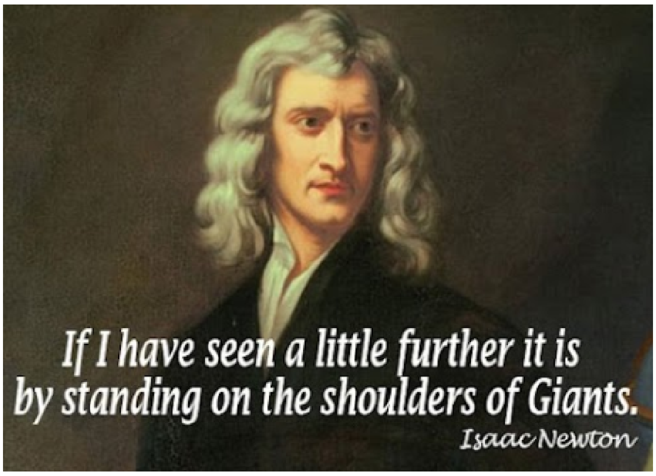
Finished files are the result of years of scientific study combined with the experience of years...



Large head of a man in the centre looking to the left, with white hair and beard.
Man in the centre left carrying a walking stick (whose head is the eye of #1)
Lady beside #2 holding a baby.
Baby in #3's arms.
Profile of woman's head above right hand column.
Mirror image of #5 above the left column.
Another face in profile on the opposite side of the #6's bird statue (a mirror of #6).
Another face in profile directly above #6, the bird forms the nose and forehead.
A face looking towards you in the extreme left, to the side of #8.







The Discovery of the Calculus

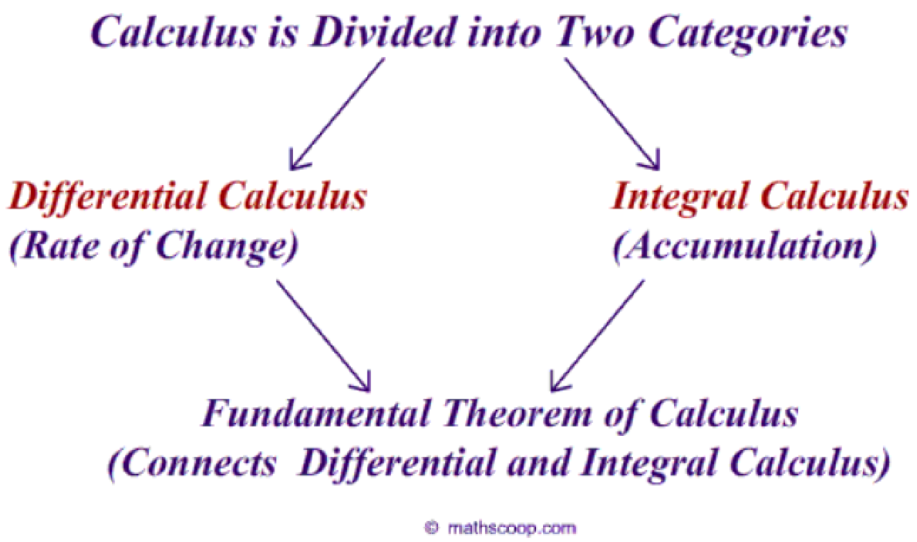
In the 1670s and 1680s, Sir Isaac Newton in England and Gottfried Leibniz in Germany, working independently, figured out the basic concepts of Calculus!

Newton wanted to have a new way to predict where to see planets in the sky, because astronomy had always been a popular and useful form of science, and knowing more about the motions of the objects in the night sky was important for navigation of ships. Leibniz wanted to work out the space (area) under a curve (a line which is not straight).

Many years later, the two men argued over who discovered it first. Scientists from England supported Newton, but scientists from the rest of Europe supported Leibniz. Most mathematicians today agree that both men share the credit equally. Some parts of modern calculus come from Newton, such as its uses in physics. Other parts come from Leibniz, such as the symbols used to write it.

So what is Calculus?
In short, Calculus is all about Δ!

Calculus is a vast topic, and it forms the basis for much of modern mathematics. There are two main branches of basic calculus; **differential calculus** and **integral calculus**.

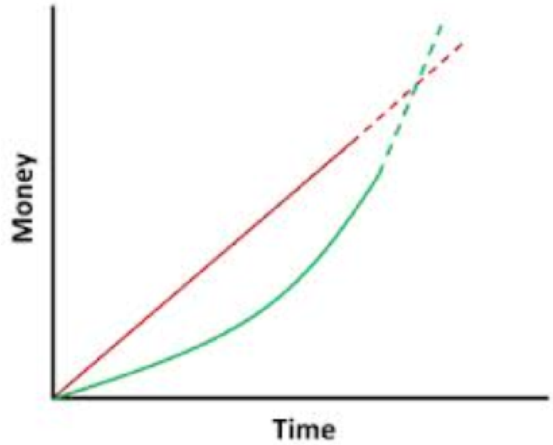


Differential calculus is the study of **rates of change of functions**. This requires finding the tangent to a curve, which requires limits. You are normally introduced to differential calculus by learning how to find the **derivative of a function** in order to <u>determine the slope of the graph of that function at any point.</u>

Integral calculus, also known as integration, is like doing a derivative in reverse. It is often introduced in terms of finding indefinite integrals and finding the area under a curve (definite integrals).

So in the world of linear functions, we can obtain the slope (rate of change, lets say the rate of money over time as an example) rather easily through the simple formula we have already used: **$\Delta y/\Delta x$.**

However, if the function is not a line, then the slope itself changes and therefore we need a new technique for finding the slope. This is called the DERIVATIVE. It is itself a function, and it allows us to find the slope at any point of the original function.



A great example to illustrate the notion of the derivative is to first consider a position function. If you are driving to Boston, then your position can be given by the function:

$$f(t) = 5t^3/3 - 25t^2 + 120t$$

where t represents the time in hours traveled and the output (y or f(t)) is the number of miles traveled.

Well, if we take the derivative of this position function, we get a velocity function! This tells us how fast we are going at any given time t.

The position function:

$$f(t) = 5t^3/3 - 25t^2 + 120t$$

has as its derivative the function:

$$v(t) = f'(t) = 5t^2 - 50t + 120$$

So...when t = time in hours...
1. How many miles have you traveled after half an hour?
2. How fast are you moving after half an hour?

ANS:
1. f(0.5) = .20833 - 6.25 + 60 = 53.958 miles traveled.
2. v(0.5) = 5/4 - 25 + 120 = 96.25mph at the half hour mark.

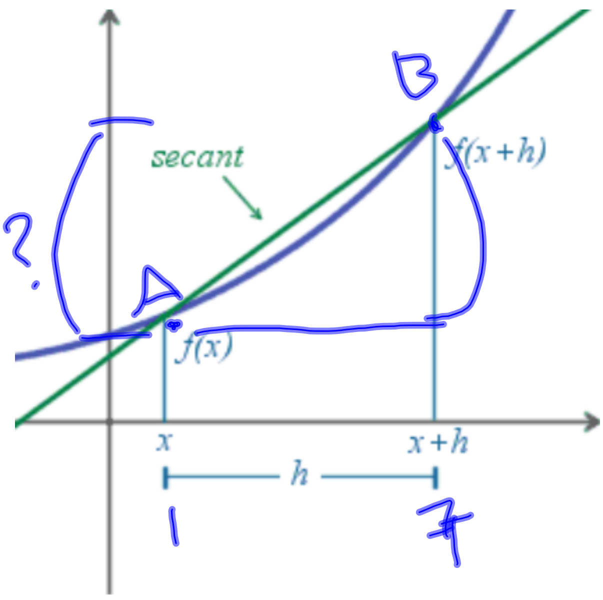
So, Calculus allows us to quantify the world in a way that algebra could never handle. It is in large part due to Calculus that we have this modern world that we so easily take for granted!

Now, as we said, the derivative of a function is another function. If the function is f(x), the derivative is generally expressed as f '(x) or y ' (we say y prime).

Now, the type of function you initially are given will determine what approach you will use to finding the derivative. There are many different rules for finding the derivative, a few key techniques are the Power Rule, Product Rule, Quotient Rule, Chain Rule.

Before we had established these rules, we had to calculate the derivative in a more manual way. This is called the **difference quotient**.
Let's walk through an example together.

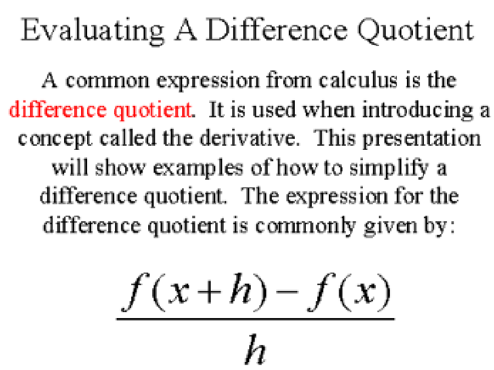
In the example to the right, we have a function f(x), given by the blue line. We want to know its slope but as you can see looking at it, the slope changes depending on where you are. So what we do is we take two points on the curve, draw a line connecting them, we call this the secant line (green line).



Now, with this secant line, we can determine the derivative using our original concept of slope, which is ΔY/ΔX. What is our ΔY/ΔX in this example?

ANS: [f(x+h) – f(x)]/H

So our slope for f(x), called the derivative, can be manually calculated through the Difference Quotient. Let's try a simple example.



If $f(x) = x^2$ then our setup looks like this:

$[(x+h)^2 - x^2]/h$

We use algebra to expand this and simplify to get:

$$f(x+h) = (x+h)^2$$
$$= x^2 + 2xh + h^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$
$$= \frac{2xh + h^2}{h}$$
$$= 2x + h$$

$$\frac{h(2x+h)}{h}$$

Find the difference quotient for the following functions:

1. $f(x) = 3 - x^2 - x$

$$\frac{3-(x+h)^2-(x+h)-(3-x^2-x)}{h}$$

2. $g(x) = x^2 + 3x$

Find the difference quotient for the following function:

2. $g(x) = x^2 + 3x$

$$\frac{(x+h)^2 + 3(x+h) - (x^2+3x)}{h}$$

$$f(x) = x$$

$$\frac{x + h - x}{h} = \frac{h}{h} = 1$$

$$f(x) = x + 2$$

$$= \frac{(x + h + 2) - 1(x + 2)}{h}$$

$$= \frac{h}{h} = 1$$

$$f(x) = x + 3$$

$$f(-3) = {}^{-}3 + 3$$

$$f(\star) = \star + 3$$

$$f(\star + h) = \star + h + 3$$

Find the difference quotient for the following functions:

1. $f(x) = 3 - x^2 - x$

$$\frac{3-(x+h)^2-(x+h)-(3-x^2-x)}{h}$$

$$3-(x^2+2xh+h^2)-x-h-3+x^2+x$$

$$\frac{-2xh-h^2-h}{h}$$

$$\frac{h(-2x-h-1)}{h}$$

$$-2x-h-1$$

$$(x+3)^2 =$$

$$(x+3)(x+3) =$$

$$x^2+3x+3x+9 =$$

$$x^2+6x+9$$

HW:

$$f(x) = x^2 - 4$$

$$g(x) = x^2 - 5x + 7$$

Discovery of Calculus

Isaac Newton (1642–1727), Gottfried Wilhelm Leibniz (1646–1716)

English mathematician Isaac Newton and German mathematician Gottfried Wilhelm Leibniz are usually credited with the invention of calculus, but various earlier mathematicians explored the concept of rates and limits, starting with the ancient Egyptians who developed rules for calculating the volume of pyramids and approximating the areas of circles.

In the 1600s, both Newton and Leibniz puzzled over problems of tangents, rates of change, minima, maxima, and infinitesimals (unimaginably tiny quantities that are almost but not quite zero). Both men understood that differentiation (finding the tangent to a curve at a point—that is, a straight line that "just touches" the curve at that point) and integration (finding the area under a curve) are inverse processes. Newton's discovery (1665–1666) started with his interest in infinite sums; however, he was slow to publish his findings. Leibniz published his discovery of differential calculus in 1684 and of integral calculus in 1686. He said, "It is unworthy of excellent men, to lose hours like slaves in the labor of calculation. . . . My new calculus . . . offers truth by a kind of analysis and without any effort of imagination." Newton was outraged. Debates raged for many years on how to divide the credit for the discovery of calculus, and, as a result, progress in calculus was delayed. Newton was the first to apply calculus to problems in physics, and Leibniz developed much of the notation seen in modern calculus books.

Today, calculus has invaded every field of scientific endeavor and plays invaluable roles in biology, physics, chemistry, economics, sociology, and engineering, and in any field where some quantity, like speed or temperature, changes. Calculus can be used to help explain the structure of a rainbow, teach us how to make more money in the stock market, guide a spacecraft, make weather forecasts, predict population growth, design buildings, and analyze the spread of diseases. Calculus has caused a revolution. It has changed the way we look at the world.

SEE ALSO Zeno's Paradoxes (c. 445 B.C.), Torricelli's Trumpet (1641), L'Hôpital's *Analysis of the Infinitely Small* (1696), Agnesi's *Instituzioni Analitiche* (1748), Laplace's *Théorie Analytique des Probabilités* (1812), and Cauchy's *Le Calcul Infinitésimal* (1823).

William Blake's Newton (1795). Blake, a poet and artist, portrays Isaac Newton as a kind of divine geometer, gazing at technical diagrams drawn on the ground as he ponders mathematics and the cosmos.

Definition:

- **Difference quotient**: is an expression of the form

$$\frac{f(a+h)-f(a)}{h}.$$

They represent the average change in the value of f between $x = a$ and $x = a + h$. They are used in calculus.

Important Properties:

- In the numerator of a difference quotient, any term that does not contain a h must subtract off.
- When evaluating functions remember that whatever is inside the parenthesis, regardless of what it looks like, is substituted into every variable x. For example, if $f(x) = x^2 - 3x + 1$ then

$$\begin{aligned} f(a+h) &= (a+h)^2 - 3(a+h) + 1 \\ &= a^2 + 2ah + h^2 - 3a - 3h + 1 \end{aligned}$$

- When evaluating a difference quotient, the h in the denominator will always divide out.

Common Mistakes to Avoid:

- Note that $f(a+h) \neq f(a) + h$.
- Remember that $(a+h)^2 \neq a^2 + h^2$. Instead,

$$(a+h)^2 = a^2 + 2ah + h^2$$

by using foil.

- Do NOT distribute inside a quantity raised to a power. Remember to raise a quantity to its power *before* you distribute. For example, $3(a+h)^2 \neq (3a+3h)^2$. Instead,

$$3(a+h)^2 = 3(a^2 + 2ah + h^2) = 3a^2 + 6ah + 3h^2.$$

PROBLEMS

Find $\frac{f(a+h)-f(a)}{h}$ for each of the given functions.

1. $f(x) = x + 2$

Here, we have that

$$f(a) = a + 2$$
$$f(a+h) = a + h + 2.$$

Substituting these into the difference quotient, we get

$$\begin{aligned} \frac{f(a+h)-f(a)}{h} &= \frac{a+h+2-(a+2)}{h} \\ &= \frac{a+h+2-a-2}{h} \\ &= \frac{h}{h} \\ &= 1 \end{aligned}$$

$$\boxed{\frac{f(a+h)-f(a)}{h} = 1}$$

---

2. $f(x) = 3x - 5$

Here, we have that

$$\begin{aligned} f(a) &= 3a - 5 \\ f(a+h) &= 3(a+h) - 5 \\ &= 3a + 3h - 5. \end{aligned}$$

Substituting these into the difference quotient, we get

$$\begin{aligned} \frac{f(a+h)-f(a)}{h} &= \frac{3a+3h-5-(3a-5)}{h} \\ &= \frac{3a+3h-5-3a+5}{h} \\ &= \frac{3h}{h} \\ &= 3 \end{aligned}$$

$$\boxed{\frac{f(a+h)-f(a)}{h} = 3}$$

3. $f(x) = x^2 - 4$

   Here, we have that

$$\begin{aligned} f(a) &= a^2 - 4 \\ f(a+h) &= (a+h)^2 - 4 \\ &= a^2 + 2ah + h^2 - 4 \end{aligned}$$

   Substituting these into the difference quotient, we get

$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{a^2 + 2ah + h^2 - 4 - (a^2 - 4)}{h} \\ &= \frac{a^2 + 2ah + h^2 - 4 - a^2 + 4}{h} \\ &= \frac{2ah + h^2}{h} \\ &= \frac{h(2a+h)}{h} \\ &= 2a + h \end{aligned}$$

$$\boxed{\frac{f(a+h) - f(a)}{h} = 2a + h}$$

---

4. $f(x) = x^2 - 5x + 7$

   We know that

$$\begin{aligned} f(a) &= a^2 - 5a + 7 \\ f(a+h) &= (a+h)^2 - 5(a+h) + 7 \\ &= a^2 + 2ah + h^2 - 5a - 5h + 7 \end{aligned}$$

   Substituting these into the difference quotient, we get

$$\begin{aligned}\frac{f(a+h)-f(a)}{h} &= \frac{a^2+2ah+h^2-5a-5h+7-(a^2-5a+7)}{h} \\ &= \frac{a^2+2ah+h^2-5a-5h+7-a^2+5a-7}{h} \\ &= \frac{2ah+h^2-5h}{h} \\ &= \frac{h(2a+h-5)}{h} \\ &= 2a+h-5\end{aligned}$$

$$\boxed{\frac{f(a+h)-f(a)}{h} = 2a+h-5}$$

---

5. $f(x) = 4x^2+3x-2$

   We know that

$$\begin{aligned}f(a) &= 4a^2+3a-2 \\ f(a+h) &= 4(a+h)^2+3(a+h)-2 \\ &= 4(a^2+2ah+h^2)+3a+3h-2 \\ &= 4a^2+8ah+4h^2+3a+3h-2\end{aligned}$$

   Substituting these into the difference quotient, we get

$$\begin{aligned}\frac{f(a+h)-f(a)}{h} &= \frac{4a^2+8ah+4h^2+3a+3h-2-(4a^2+3a-2)}{h} \\ &= \frac{4a^2+8ah+4h^2+3a+3h-2-4a^2-3a+2}{h} \\ &= \frac{8ah+4h^2+3h}{h} \\ &= \frac{h(8a+4h+3)}{h} \\ &= 8a+4h+3\end{aligned}$$

$$\boxed{\frac{f(a+h)-f(a)}{h} = 8a+4h+3}$$

6. $f(x) = \dfrac{1}{x}$

   Here, we have that

$$f(a) = \frac{1}{a}$$

$$f(a+h) = \frac{1}{a+h}$$

   Substituting these into the difference quotient, we get

$$\frac{f(a+h)-f(a)}{h} = \frac{\dfrac{1}{a+h} - \dfrac{1}{a}}{h}$$

$$= \frac{\dfrac{a}{a(a+h)} - \dfrac{(a+h)}{a(a+h)}}{h}$$

$$= \frac{\dfrac{a-(a+h)}{a(a+h)}}{h}$$

$$= \frac{\dfrac{a-a-h}{a(a+h)}}{h}$$

$$= \frac{\dfrac{-h}{a(a+h)}}{h}$$

$$= \frac{-h}{ah(a+h)}$$

$$= \frac{-1}{a(a+h)}$$

$$\boxed{\frac{f(a+h)-f(a)}{h} = \frac{-1}{a(a+h)}}$$

7. $f(x) = \dfrac{3}{x+2}$

Here, we have that

$$f(a) = \frac{3}{a+2}$$

$$f(a+h) = \frac{3}{a+h+2}$$

Substituting these into the difference quotient, we get

$$\begin{aligned}
\frac{f(a+h)-f(a)}{h} &= \frac{\dfrac{3}{a+h+2} - \dfrac{3}{a+2}}{h} \\
&= \frac{\dfrac{3(a+2)}{(a+2)(a+h+2)} - \dfrac{3(a+h+2)}{(a+2)(a+h+2)}}{h} \\
&= \frac{\dfrac{3(a+2)-3(a+h+2)}{(a+2)(a+h+2)}}{h} \\
&= \frac{\dfrac{3a+6-3a-3h-6}{(a+2)(a+h+2)}}{h} \\
&= \frac{\dfrac{-3h}{(a+2)(a+h+2)}}{h} \\
&= \frac{-3h}{h(a+2)(a+h+2)} \\
&= \frac{-3}{(a+2)(a+h+2)}
\end{aligned}$$

$$\boxed{\frac{f(a+h)-f(a)}{h} = \frac{-3}{(a+2)(a+h+2)}}$$

Lesson 25 Summary: April 25th
Venn Diagrams: Revisit
Law of Large Numbers
(Presentations)

Lesson Handouts
Mobius Strip Instructions
Venn Diagrams (Cliff Pickover, The Math Book)
Infinite Monkey Theorem (Cliff Pickover, The Math Book)
Googol (Cliff Pickover, The Math Book)

WARM UP:

1. If a velocity function is defined by v(t) = f'(t) = $5t^2$ - 50t + 120, where t is time in hours, find the speed at 90 minutes.

2. Calculus is the study of _______.

3. What is the derivative?

4. Write out the Difference Quotient.

5. Write the difference quotient for f(x) = $4x^2$ + 2x - 1

6. Simplify #5.

WARM UP:

1. If a velocity function is defined by $v(t) = f'(t) = 5t^2 - 50t + 120$, where t is time in hours, find the speed at 90 minutes.

1.5

2. Calculus is the study of __Δ___.

3. What is the derivative?

4. Write out the Difference Quotient.

$$\frac{f(x+h) - f(x)}{h}$$

5. Write the difference quotient for $f(x) = 4x^2 + 2x - 1$

6. Simplify #5.

$$\frac{4(x+h)^2 + 2(x+h) - 1 - (4x^2 + 2x - 1)}{h}$$

$$\frac{4(x+h)^2 + 2(x+h) - 1 \quad (4x^2 + 2x - 1)}{h}$$

$4(x^2+2xh+h^2)+3x+2h-1$

$-4x^2-3x+1$

$$\frac{8xh+4h^2+2h}{h}$$

$$\frac{h(8x+4h+2)}{h}$$

$(x+h)^2 =$

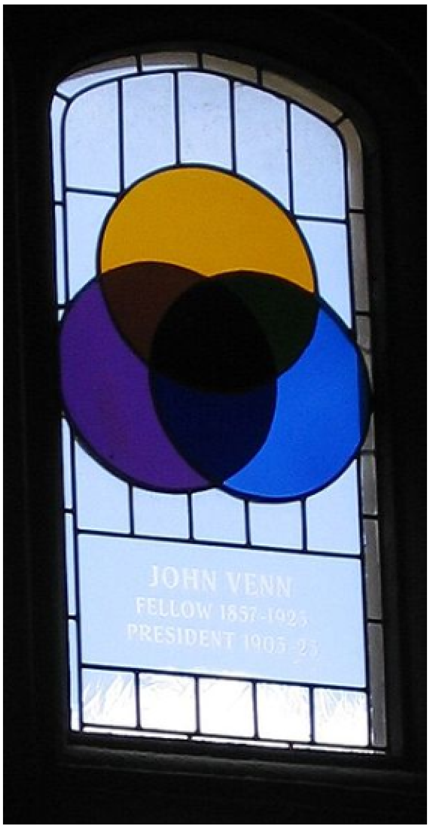
$(x+h)(x+h) =$

$x^2+1xh+1hx+h^2$

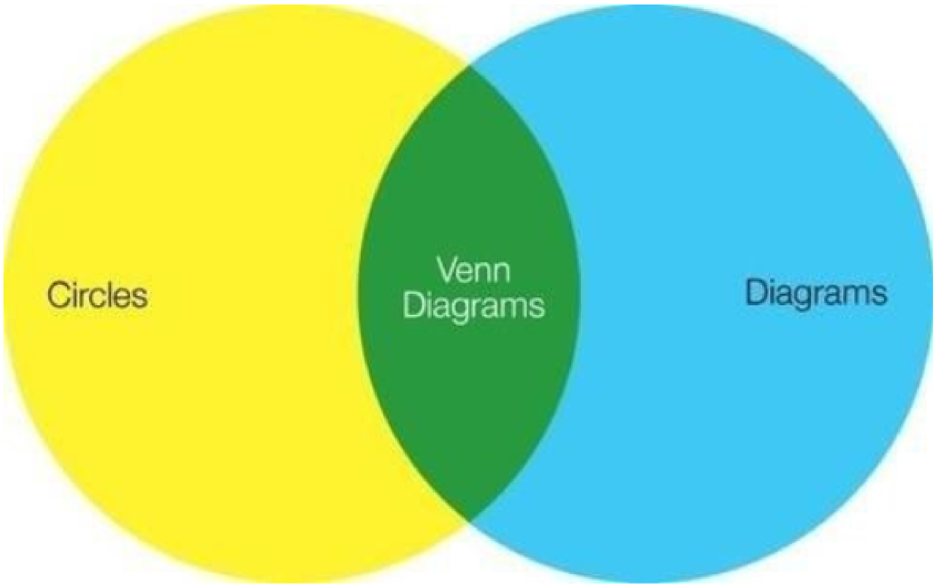
$4x^3 + 2x - 1$

While we often think of math in terms of symbols, abstraction, and complicated looking formulas, in fact, much of mathematics is understood visually, just consider Geometry.

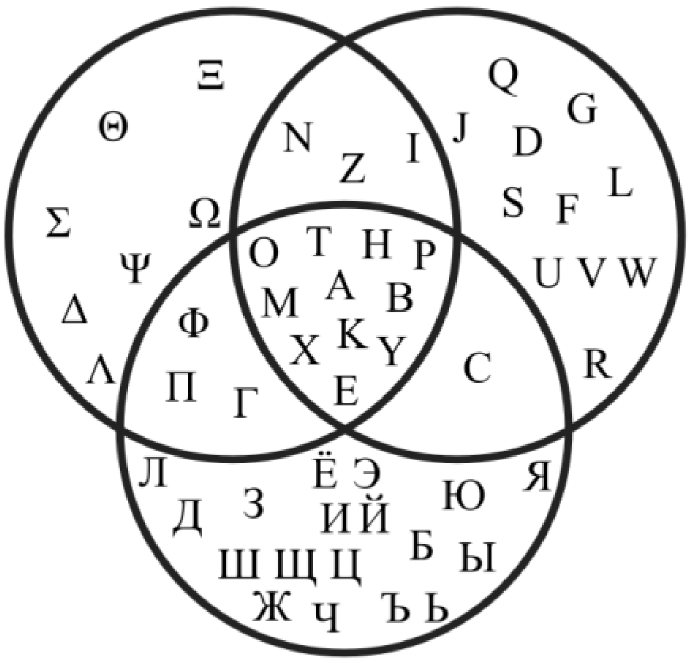
One major example of the visual being utilized to understand logical relationships is **Venn Diagrams (1880)**. We have already spent some time considering these types of problems, they are excellent in visualizing groups of items and the relationships between items.
Many problems in probability, statistics, and computer science make heavy use of Venn Diagrams.



Visual Fun for the Day:

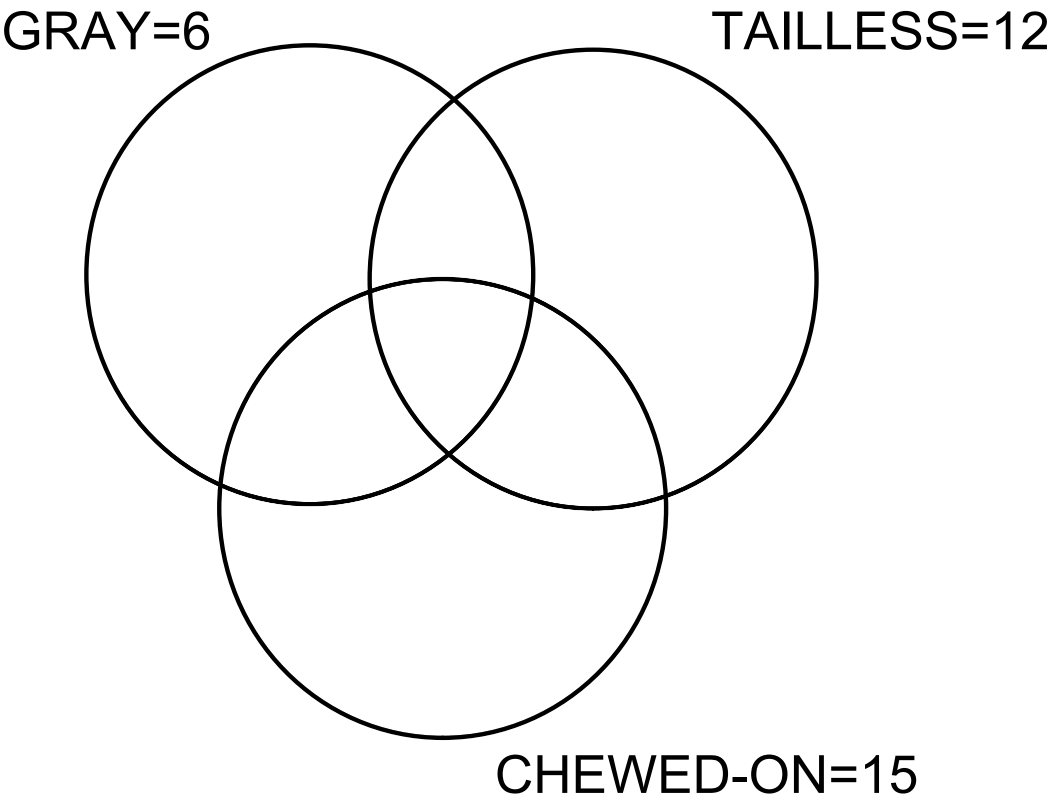


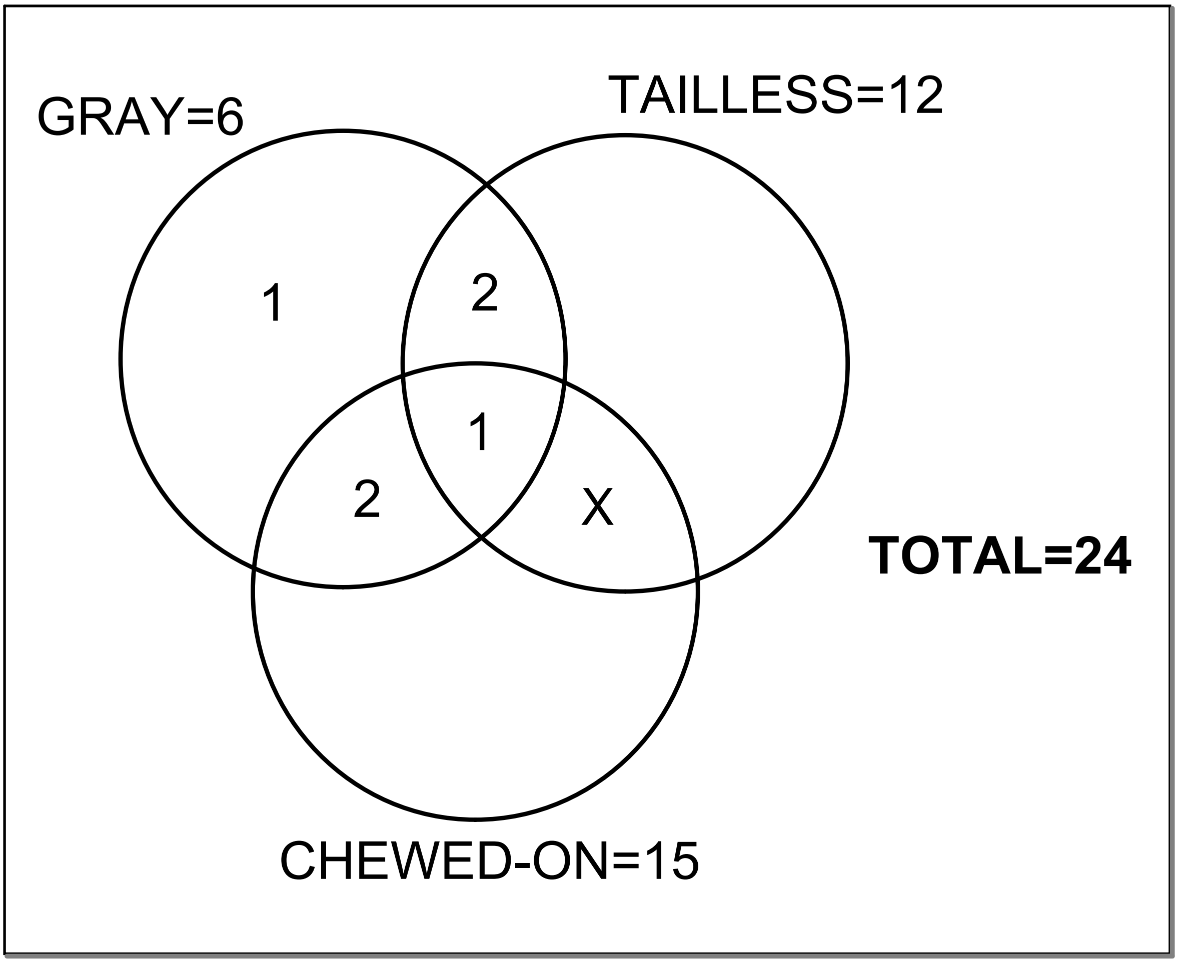
What is this Venn diagram illustrating?

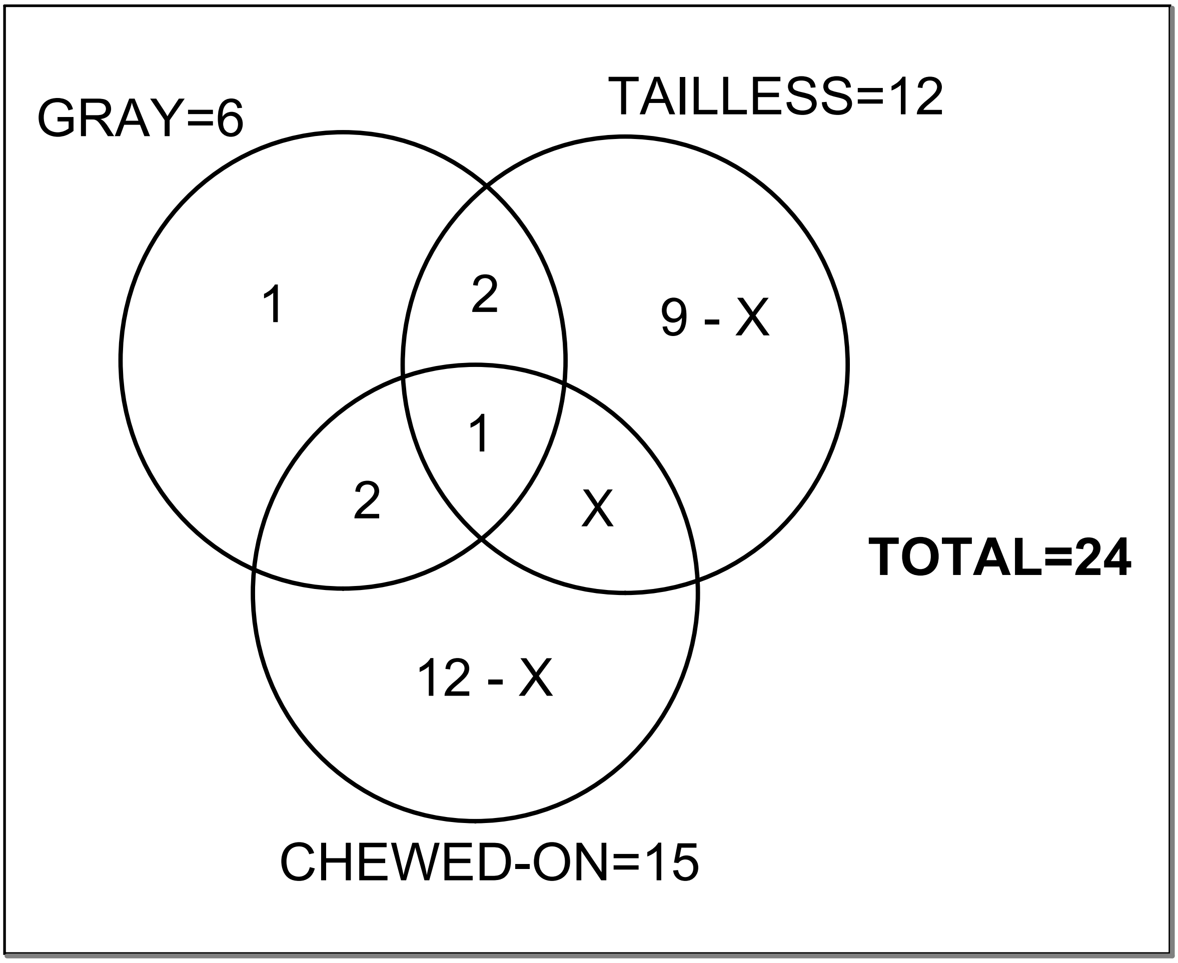


The Greek, Latin and Russian alphabets

Suppose I discovered that my cat had a taste for the adorable little geckoes that live in the bushes and vines in my yard, back when I lived in Arizona. In one month, suppose he deposited the following on my carpet:
six gray geckoes,
twelve geckoes that had dropped their tails in an effort to escape capture,
and **fifteen geckoes that he'd chewed on a little.**
Only one of the geckoes was gray, chewed on, and tailless;
two were gray and tailless but not chewed on;
two were gray and chewed on but not tailless.
If there were a **total of 24 geckoes left on my carpet that month**, and all of the geckoes were at least one of "gray", "tailless", and "chewed on", <u>how many were tailless and chewed on but not gray?</u>




GRAY=6
TAILLESS=12
1
2
1
2
X
TOTAL=24
CHEWED-ON=15


GRAY=6
TAILLESS=12
1
2
9 - X
1
2
X
TOTAL=24
12 - X
CHEWED-ON=15

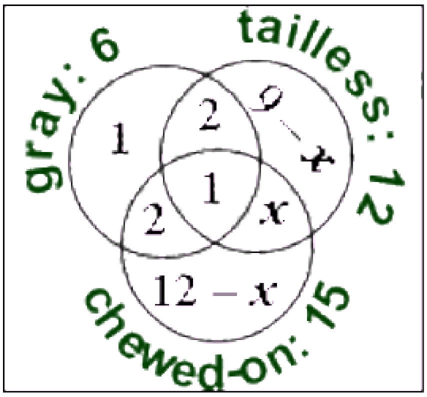
ANS:

$1 + 2 + 1 + 2 + x + (12 – x) + (9 – x) = 24$

$27 - x = 24$

$x = -3$

$\mathbf{x = 3}$



Three geckoes were tailless and chewed on but not gray.

NOTE: No geckoes were injured during the production of this math problem.

In probability theory, the **law of large numbers (LLN)** is a theorem that describes the result of performing the same experiment a large number of times. According to the law, the average of the results obtained from a large number of trials should be close to the **expected value**, and will tend to become closer as more trials are performed. For example, as the number of coin flips approaches infinity, so too will the odds of getting a head approach 1/2. This is where the "observable" probability meets up with the "theoretical" probability.

The LLN is important because it "guarantees" stable long-term results for the averages of random events. For example, while a casino may lose money in a single spin of the roulette wheel, its earnings will tend towards a predictable percentage over a large number of spins. Any winning streak by a player will eventually be overcome by the parameters of the game.

It is important to remember that the LLN only applies (as the name indicates) when a large number of observations are considered. There is no principle that a small number of observations will coincide with the expected value or that a streak of one value will immediately be "balanced" by the others.

What is the expected value of a single roll of a die according to the LLN?
Remember, each side has equal probability of 1/6, but each side has different values.

HINT: What is the expected value of a die that has all sides 1?

HINT2: It is not an integer.

ANS: (1/6)*1 + (1/6)*2 + (1/6)*3 + (1/6)*4 + (1/6)*5 + (1/6)*6 = (1+2+3+4+5+6)/6 = 21/6 = 3.5. The average expectation of a die roll as we roll the die an infinite number of times is **3.5**.

For Fun:

1. Create a one sided surface out of a regular piece of paper.

2. Read about monkeys typing out all the works of Shakespeare.

3. The origins of how the most popular search engine of the planet was named.

Googol

PRESENTATIONS (last 20 minutes of class)

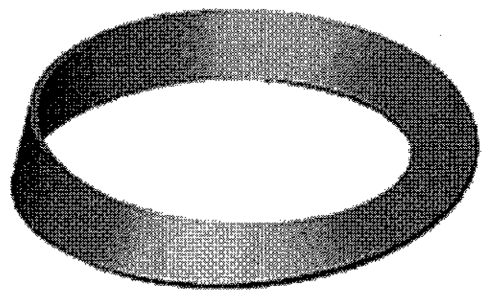
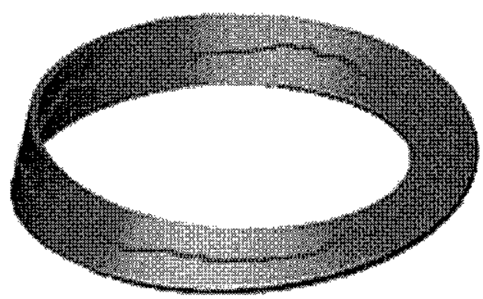
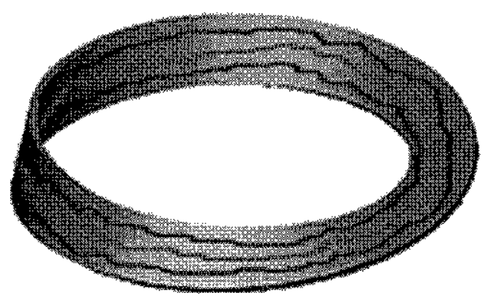
Ryan - History of Math Technology

Marlena - Interesting math concepts

Mobius Strip

Look at a normal sheet of paper. Count the number of sides. There's two - a front and a back, right? Pretty typical.

Here's how you make a not-so-typical piece of paper.

Take a long, thin strip of paper... 4 cm by 24 cm, for example. Anything reasonably close will do. Give one end a half twist and then tape it together. It should look similar to this: **(except it doesn't have to be blue.)**	
What you now have is a one-sided, one-edged piece of paper called the Mobius strip. To see that it has only one side, try drawing a line along one side and continue until you end up where you started. Like this: **Now see if you can find the "other side"... the side without a line on it. You won't find the other side, because there isn't one!**	
To see that there is only one edge, use a different colored pencil or pen and draw a line along the edge on one side of the center line you drew. Eventually you will end up where you started, and you will see that there are lines along "both" edges of the Mobius strip. **But since you drew only one continuous line, there must be only one edge!**	

Now think about what would happen if you cut down the center of your Mobius strip. An ordinary paper ring cut in half would give you two seperate rings, right? However, if you cut down the center of a Mobius strip, instead you end up one ring twice as large as the original!

Infinite Monkey Theorem

Félix Édouard Justin Émile Borel (1871–1956)

The infinite monkey theorem states that a monkey pressing keys at random on a typewriter keyboard for an infinite amount of time will almost surely type a particular finite text, such as the Bible. Let us consider a single biblical phrase, "In the beginning God created the heavens and the earth." How long would it take a monkey to type this phrase? Assume that there are 93 symbols on a keyboard. The phrase contains 56 letters (counting spaces and the period at the end). If the probability of hitting the correct key on the typewriter is $1/n$, where n is the number of possible keys, then the probability of the monkey correctly typing 56 consecutive characters in the target phrase is, on average, $1/93^{56}$, which means that the monkey would have to try more than 10^{100} times, on average, before getting it right! If the monkey pressed one key per second, he'd be typing for well over the current age of the universe.

Interestingly, if we were to save characters that are typed correctly, the monkey would obviously require many fewer keystrokes. Mathematical analysis reveals that the monkey, after only 407 trials, would have a 50/50 chance that the correct sentence was typed! This crudely illustrates how evolution can produce remarkable results when harnessing nonrandom changes by preserving useful features and eliminating nonadaptive ones.

French mathematician Émile Borel mentioned the "dactylographic" (that is, typewriting) monkeys in a 1913 article, in which he commented on the likelihood of one million monkeys typing 10 hours a day to produce books in a library. The physicist Arthur Eddington wrote in 1928, "If an army of monkeys were strumming on typewriters, they *might* write all the books in the British Museum. The chance of their doing so is decidedly more favorable than the chance of [all gas molecules in a vessel suddenly moving to] one half of the vessel."

SEE ALSO Law of Large Numbers (1713), Laplace's *Théorie Analytique des Probabilités* (1812), Chi-Square (1900), and The Rise of Randomizing Machines (1938).

According to the infinite monkey theorem, a monkey pressing keys at random on a typewriter keyboard for an infinite amount of time will almost surely type a particular finite text, such as the Bible.

Googol

Milton Sirotta (1911–1981), Edward Kasner (1878–1955)

The term *googol,* which stands for the number 1 followed by 100 zeros, was coined by nine-year-old Milton Sirotta. Milton and his brother Edwin worked for most of their lives in their father's factory in Brooklyn, New York, pulverizing apricot pits to form an abrasive used for industrial purposes. Sirotta was the nephew of American mathematician Edward Kasner, who popularized the term after he asked Milton to make up a word for a very large number. The word *googol* first appeared in print publications in 1938.

Kasner is famous for being the first Jew appointed to a faculty position in the sciences at Columbia University and for his coauthoring of the book *Mathematics and the Imagination*, in which he introduced *googol* to a wide nontechnical audience. Although googol is of no special significance in mathematics, it has proven to be very useful for comparing large quantities, and for stimulating awe in the public mind as to the wonders of mathematics and the vast universe in which we live. It has also changed the world in other ways. Larry Page, one of the founders of the Internet search engine Google, was intrigued by mathematics and named his company after googol, after accidentally misspelling the word.

A little more than a googol different ways exist to arrange 70 items in a sequence, such as 70 people waiting in line to enter a doorway. Most scientists agree that if we could count all the atoms in all the stars we can see, we would have far less than a googol atoms. A googol years are required for all the black holes in the universe to evaporate. However, the number of possible chess games is *more* than a googol. The term *googolplex* is 1 followed by a googol number of zeros. It has more *digits* than there are atoms in stars in the visible universe.

SEE ALSO Archimedes: Sand, Cattle & Stomachion (c. 250 B.C.), Cantor's Transfinite Numbers (1874), and Hilbert's Grand Hotel (1925).

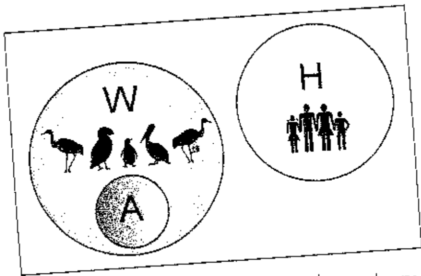
A little more than a googol different ways exist for arranging the 70 beads in sequence, assuming that each bead is different and that the necklace remains open.

Venn Diagrams

John Venn (1834–1923)

In 1880, John Venn, a British philosopher and cleric in the Anglican Church, devised a scheme for visualizing elements, sets, and logical relationships. A *Venn diagram* usually contains circular areas representing groups of items sharing common properties. For instance, within the universe of all real and legendary creatures (the bounding rectangle in the first illustration), region *H* represents the humans, region *W* the winged creatures, and region A the angels. A glance at the diagram reveals that: (1) All angels are winged creatures (region A lies entirely within region W); (2) No humans are winged creatures (regions *H* and *W* are nonintersecting); and (3) No humans are angels (regions *H* and A are nonintersecting).

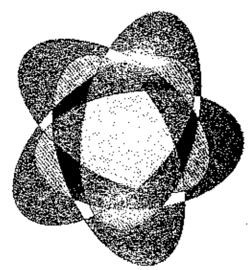
This is a depiction of a basic rule of logic—namely, that from the statements "all A is W" and "no *H* is W," it follows that "no *H* is A." The conclusion is evident when we look at the circles in the diagram.



1

The uses of these kinds of diagrams in logic were used before Venn—for example, by mathematicians Gottfried Leibniz and Leonhard Euler—but Venn was the first to comprehensively study them and formalize and generalize their usage. In fact, Venn struggled with generalizing *symmetrical* diagrams for visualizing more sets with intersecting areas, but he only got as far as 4 sets using ellipses.

A century passed before Branko Grünbaum, a mathematician at the University of Washington, showed that rotationally symmetric Venn diagrams can be made from 5 congruent ellipses. The second illustration shows one of many different symmetrical diagrams for 5 sets.


2

Mathematicians gradually realized that rotationally symmetric diagrams can be drawn with prime numbers of petals only. However, symmetrical diagrams with 7 petals were so hard to find that mathematicians initially doubted their existence. In 2001, mathematician Peter Hamburger and artist Edit Hepp constructed an example for 11 petals, shown on the opposite page.

SEE ALSO Aristotle's *Organon* (c. 350 B.C.), Boolean Algebra (1854), *Principia Mathematica* (1910–1913), Gödel's Theorem (1931), and Fuzzy Logic (1965).

Symmetric 11-Venn diagram, courtesy of Dr. Peter Hamburger and Edit Hepp.

<u>Lesson 26 Summary: April 30th</u>

Homage to Shakuntala Devi

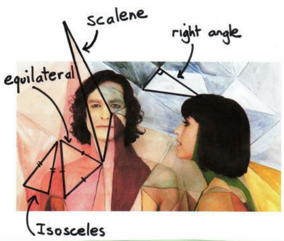
Binary

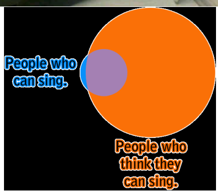
<u>Lesson Handouts</u>

The Magic of Binary (Article)

Binary Number Systems (Worksheet)

SEE, MATH INFLUENCES POPULAR MUSIC!





SOME TRIANGLES THAT I USED TO KNOW

Apr 30-11:19 AM

WARM UP: $\frac{f(x+h)-f(x)}{h}$

1. Figure out the derivative using the DQ for f(x) = $4x^2 + 3x - 1$

2. What is a Venn diagram? Draw an original example.

3. If the expected value for a die roll is 3.5, what is the expected value for a die that only has 4 sides? HINT: This die has sides of 1,2,3,4.

4. What is special about the Mobius Strip?

If two cards are drawn without replacement from an ordinary deck, find the probabilities of the following results.

9. The second is a heart, given that the first is a heart.

10. The second is black, given that the first is a spade.

11. The second is a face card, given that the first is a jack.

Feb 6-9:21 AM

WARM UP: $\frac{f(x+h)-f(x)}{h}$

1. Figure out the derivative using the DQ for f(x) = $4x^2 + 3x - 1$

$$\frac{4(x+h)^2+3(x+h)-1-(4x^2+3x-1)}{h}$$

$$4x^2+8xh+4h^2+3x+3h-1-4x^2-3x+1$$

$$\frac{h(8x+4h+3)}{h}$$

Feb 6-9:21 AM

Find a timer on your phone or watch and without a calculator, time yourself to see how long it takes you to answer the following problem. Your goal is to finish as quickly as possible without a mistake!

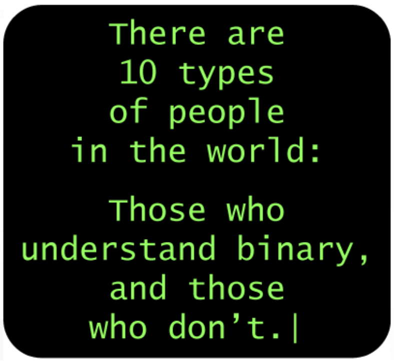
Add the following numbers:
25,842,278
111,201,721
370,247,830
55,511,315
Then multiply the result by 9878.

Did you get 5,559,369,456,432?

Rest in Peace Shakuntala Devi

This is an actual problem that she solved in less than 20 seconds in her head.

Apr 30-11:33 AM



Apr 30-11:13 AM

<u>Binary: A Truly Modern Number System</u>

One aspect of mathematics that we internalize early, then forget about is the digits we use for counting.

They are:

0,1,2,3,4,5,6,7,8,9

The speculation is that since we have 10 fingers and toes, we ended up with 10 digits for counting. The key point here is that <u>we are not required to use 10 digits</u> for counting, we can use whatever we like...

Apr 30-10:45 AM

Just consider...

Standard positional numeral systems

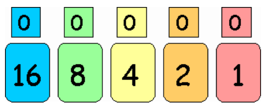
The common names are derived somewhat arbitrarily from a mix of Latin and Greek.[citation needed]

Base	Name	Usage
2	Binary	Digital computing
3	Ternary	Cantor set (all points in [0,1] that *can* be represented in ternary with no 1s); counting Tasbih in Islam; hand-foot-yard and teaspoon-tablepoon-shot measurement systems; most *economical* integer base system
4	Quaternary	Data transmission and Hilbert curves; Chumashan languages, Ventureño language, and Kharosthi numerals
5	Quinary	Gumatj language, Nunggubuyu language, Kuurn Kopan Noot language, and Saraveca
6	Senary	Diceware, Ndom language, Proto-Uralic (suspected)
7	Septenary	Week cycle
8	Octal	Charles XII of Sweden, Unix-like permissions, DEC PDP-11
9	Nonary	
10	Decimal	Most widely used by modern civilizations[1][2][3]
11	Undecimal	Jokingly proposed during the French revolution to settle a dispute between those proposing a shift to duodecimal and those who were content with decimal
12	Duodecimal	Languages in the Nigerian Middle Belt such as Janji, Gbiri-Niragu (Kahugu), the Nimbia dialect of Gwandara
13	Tridecimal	A cycle of the Maya calendar
14	Tetradecimal	Programming for the HP 9100A/B calculator[4] and image processing applications.[5]
15	Pentadecimal	Telephony routing over IP, and the Huli language
16	Hexadecimal	Base16 encoding; compact notation for binary data
18	Octodecimal	A cycle of the Mesoamerican Long Count calendar
20	Vigesimal	Celtic numerals, Maya numerals, Yoruba numerals, Tlingit people, Dzongkha numerals, Santali language, Ainu language
24	Tetravigesimal	Umbu-Ungu also known as Kakoli.
25	Pentavigesimal	Compact notation of quinary numbers
26	Hexavigesimal	
27	Septemvigesimal	Telefol and Oksapmin languages, and compact notation of ternary numbers
30	Trigesimal	Month cycle for various calendars
32	Duotrigesimal	Base32 encoding, and the Ngiti language
36	Hexatrigesimal	Base36 encoding, and compact notation of senary numbers
60	Sexagesimal	Babylonian numeral system; degrees-minutes-seconds and hours-minutes-seconds measurement systems
64	Tetrasexagesimal	Base64 encoding; compact notation of quaternary or of octal numbers

Apr 30-10:56 AM

It turns out that the system of counting using only two digits (0 and 1) allowed us to create a whole ***<u>new plane of existence</u>***, namely the Digital World!

Let's learn how to convert from our human way of looking at numbers like 42 to the digital way of looking at numbers like 101010.



Apr 30-10:59 AM

To convert from Decimal to Binary, consider:

2^7					2^2	2^1	2^0
128	64	32	16	8	4	2	1
1	0	0	1	1	0	1	1

What is this number?

Apr 30-12:23 PM

Before we dive into the worksheet, start by converting your favorite number AND our 10 digits of counting into their respective Binary counterparts.

0 = 0
1 = 1
2 = 10
3 = 11
4 = 100
5 = 101
6 = 110
7 = 111
8 = 1000
9 = 1001

			1	0	1
2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1

Apr 30-11:19 AM

GRADE UPDATE

Come up one at a time and we can review your grade status as we head into the final.

You may leave after we have talked.

Apr 30-11:50 AM

The Magic of Binary

Thu, 3rd Feb 2011

How do two digits - 0 and 1 - change the world?

Jeff Zilahy

"Any sufficiently advanced technology is indistinguishable from magic."

Arthur C. Clarke

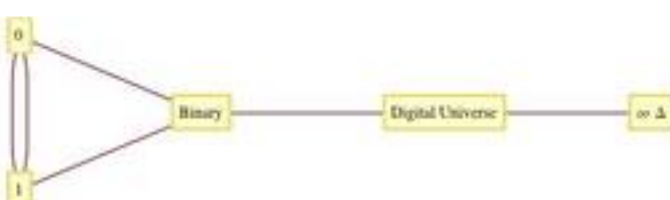
Have you ever wondered how the Internet, computers and video games happen? How exactly does your computer manage to display the words you are reading right now? Does it feel a bit like magic?

To know the secret underlying the entire digital world, whether it is your iPod with 10,000 of your favourite songs or Facebook.com or that new Android phone, is to know nothing more than a series of 0's and 1's. All of this technology is just a wonderful, nay, genius, application of good old maths.

While our precious Earth has existed for billions of years, and we *Homo sapiens* have been plodding about for something like 100,000 years, computers did not even exist a century ago. So what on Earth caused this dramatic and epic digital revolution? The short answer to that question is the binary system.

To make sense of the binary system, it helps to briefly revisit a very popular system related to binary, that being the decimal system. The decimal system is what most modern civilizations use to make numbers. When you think of any number, for example 42 or 0.007, what numerical ingredients did you use to make these numbers? The answer is rather trivially the 10 digits of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. It doesn't matter what number we are talking about, you must use that pool of 10 digits. Some people surmise that the reason we use 10 digits is because we ourselves have 10 digits, namely our fingers and our toes.

What is an often overlooked fact is that we do not have to use 10 digits to build a number system. It is a rather arbitrary decision, really, to use 10 digits. In fact, many civilizations throughout history toyed with using a different number of digits to build numbers. The most relevant of these systems to our story is binary, where we use a measly 2 digits to build numbers.

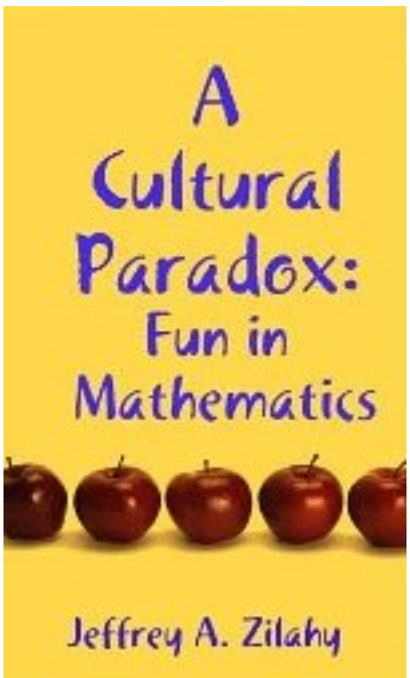


So, the digits of binary are limited to 0 and 1. You may be asking, how am I meant to build any number with just those two choices?

Pretend we have some random and familiar number, say 23. How would we represent that number if we only have a 0 and a 1 to choose from? Well, the key is to think of the powers of two.

As any elementary exponents lesson will tell you, some initial powers of two are: 2^0, 2^1, 2^2, 2^3, 2^4, 2^5. These numbers are really just 1, 2, 4, 8, 16, 32. Now, the key to building numbers in binary is to consider each of these powers of two to be either "on" or "off". The way we indicate on is with a 1 and we indicate off with a 0. So, when we want to obtain our chosen number, we first determine the sum of powers of two that yields 23. This is 16 + 4 + 2 + 1. Now, all that we have left to do to write this number in binary is to turn on all the slots that we need, namely 16, 4, 2 and 1, and to write this number in that particular order. This means that the 2^4 slot is on, but the 2^3 is off, 2^2 is on, 2^1 is on and finally the 2^0 is also on. When we write this number out, it yields the translation of our familiar number 23 to 10111 in binary.

While we humans might have the luxury of being able to interpret and understand different number systems, modern computers only understand two states, namely our 0 and 1. This is where the binary system works exceedingly well, since computers excel at interpreting binary values.



In conjunction with a system of logic that underlies all computers called Boolean algebra, named after the mathematician George Boole, we can assign to each binary number a symbol or instruction. Then, when the computer is fed a string of binary numbers, it is able to decode those numbers into letters, or symbols or instructions that allow the creation of all that you see on your computer screen.

The digital world certainly shows no sign of abating, everyday becoming more complex, more layered, even more human. As our lives become ever more intertwined with this digital realm, perhaps this technological evolution is not that surprising after all. The notion of binary, of 0's and 1's, of two parts, really is inherent in all we know. The sum total of all human experience exists within a duality, just consider: male and female, day and night, black and white, love and hate, hot and cold, finite and infinite. The world as we can only hope to know it is as simple as the yin and the yang, a truly binary experience.

Jeff Zihaly has written a book about the magic of maths entitled *A Cultural Paradox: Fun in Mathematics*. Find it here on Google books and Amazon.

BINARY NUMBER SYSTEM WORKSHEET:

1. Convert the following binary numbers to their decimal equivalents.

a) 111_2

b) 1100001_2

c) 11000_2

d) 11110_2

e) 10110_2

f) 11101_2

g) 110101_2

h) 1001011_2

i) 10_2

j) 1110_2

k) 10111_2

l) 100001_2

2. Convert the following decimal numbers to their binary equivalents.

a) 7_{10}

b) 19_{10}

c) 58_{10}

d) 123_{10}

e) 195_{10}

f) 104_{10}

g) 96_{10}

h) 203_{10}

i) 49_{10}

j) 91_{10}

k) 1029_{10}

l) 2984_{10}

Binary to Decimal Conversion Worksheet

Name:

10101001	=	
00110010	=	
00111000	=	
01100010	=	
11101110	=	
11100001	=	
00101101	=	
00011000	=	
11010110	=	
01110010	=	

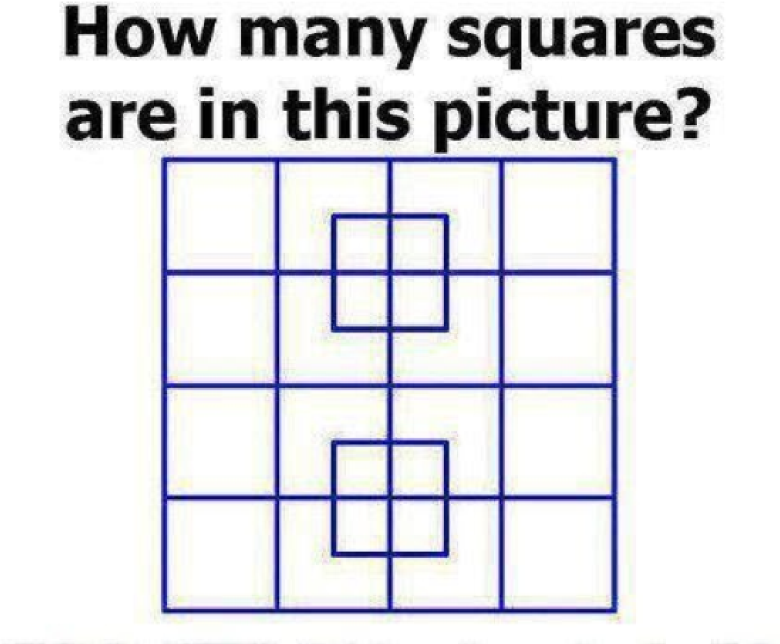
Lesson 27 Summary: May 2nd
Explanation of Computer Based Lesson
Wolfram Alpha

Lesson Handouts
Wolfram Alpha 30 question assignment

Lesson 28 Summary: May 7th
Final Review
Lesson Handouts
Final Review Tips

Lesson 29 Summary: May 9th
FINAL

FOR FUN



ANS: 40

FINAL EXAM INFORMATION

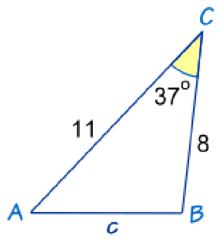
THURSDAY MAY 9TH

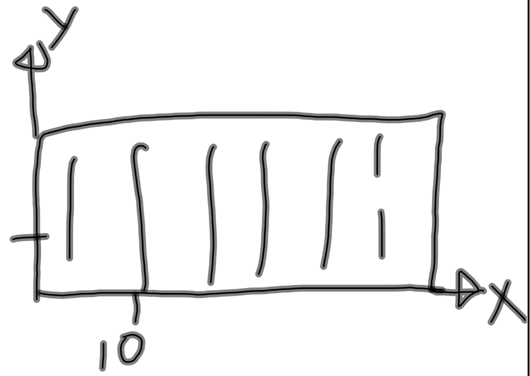
12:20 pm - 2:20 pm

WARM UP/FINAL PRACTICE:

1. What is the common ratio r in the geometric sequence .5, 1.5, 4.5, 13.5, 40.5,… ?

2. How long is side C?





3. Simplify $\sqrt{-100}$

4. In a football game, the quarterback throws a pass from the 10-yard line, 12 yards from the sideline. The pass is caught on the 45-yard line, 40 yards from the same sideline. How long was the pass?

5. Use the difference quotient to determine the derivative of $f(x) = 3x^2 + 4x - 2$.

6. Convert the following numbers into binary: 2, 10, 42, 101.

ANSWERS:

1. R = 3

2. We know angle C = 37º, a = 8 and b = 11
The Law of Cosines says: c2 = a2 + b2 - 2ab cos(C)
Put in the values we know: c2 = 82 + 112 - 2 × 8 × 11 × cos(37º)
Do some calculations: c2 = 64 + 121 - 176 × 0.798…
Which gives us: c2 = 44.44...
Take the square root: c = √44.44 = 6.67 (to 2 decimal places)
Answer: c = 6.67

3. $\sqrt{-1} * \sqrt{100} = 10i$

4. Ordered Pairs are (10,12), (45,40), so we get...

$$d = \sqrt{(45-10)^2 + (40-12)^2} = \sqrt{1225 + 784} = \sqrt{2009} \approx 44.8$$

5. $[(3(x+h)^2 + 4(x+h) - 2 - (3x^2 + 4x - 2)]/h =$
$[3x^2 + 6xh + 3h^2 + 4x + 4h - 2 - 3x^2 - 4x + 2]/h =$
$[6xh + 3h2 + 4h]/h = h(6x + 3h + 4)/h = 6x + 3h + 4 = 6x+4$

6. 10, 1010, 101010, 1100101

BASE 3 → (0, 1, 2)

33 =

1	0	0	0	0	1
2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1

1 0 1 1 1

1) $\dfrac{3(x+h)^2+4(x+h)-2-(3x^2+4x-2)}{h} =$

$\dfrac{3(x^2+2xh+h^2)+4x+4h-2-3x^2-4x+2}{h} =$

$\dfrac{3x^2+6xh+3h^2+4x+4h-2-3x^2-4x+2}{h} =$

$\dfrac{6xh+3h^2+4h}{h} \rightarrow \dfrac{h(6x+3h+4)}{h} =$

$6x+3h+4$

MAT 108

March 7, 2013

MIDTERM:

TIPS FOR SUCCESS

In order to succeed on the midterm (March 12th, no makeups!) , you should be confident with the following topics, concepts and ideas. Any questions you may have are your responsibility to seek clarification on, please ask! (jzilahy@mc3.edu). The PDF files of the smartnotes will be your best source for preparation. The following topics are an overview and may or may not appear on the actual midterm, and any content that we covered to date in class is fair game.

You should be able to know and use the following:

Quadratic Formula

Rate x Time = Distance

Heron's Formula

Rules of Exponents

Basic probability problems

Pythagorean Theorem

Area of a Triangle Formula

Prime numbers (definition, factorization, and the Sieve of Erasthosthenes)

Review the proofs for 2=1, $x^0=1$

Add, subtract, multiply and divide fractions

Historical events and people covered so far

MCCC

Spring 2013

MAT108: MIDTERM

each problem is worth 5 points

1. Give an example of an ancient mathematical artifact.

2. List the digits in the decimal system, and list the digits in the binary system.

3. Write the numbers 42 and 99 in the Roman numeral system. (see table)

4. Give a specific example of how animals/non-humans use mathematics.

5. You are given the fractions 3/4 and 5/7. Find their sum, difference, product and quotient.

6. What is a unit fraction? What ancient civilization struggled with unit fractions? Express 3/4 as the sum of two unit fractions.

7. Simplify into a single fraction: 1/a + 1/b + 1/c

8. Define prime number. Give at least two examples of prime numbers that are greater than 100.

9. Write the number 345 in prime factorization format.

10. Given a right triangle with a side of length 65 and hypotenuse of length 97, what is the length of the missing side?

11. Given a triangle with area of 125 and a height of 10, what is its base length?

12. What is the probability of getting an ace, red king, OR the queen of diamonds when selecting a single card out of a regular deck of 52 cards?

13. Where is the mistake in the proof that 2=1? (see proof)

14. Name two major historical math books that we have discussed so far in class.

15. What is the definition of an irrational number? Give an example.

16. Mark traveled to the courthouse and back. The trip there took five hours and the trip back took four hours. He averaged 35 km/h on the return trip. Find the average speed of the trip there.

17. Solve $4x^2 + 9x + 2 = 0$ using the Quadratic Formula.

18. Simplify the following exponent problem.

$(3^{-1}a^4b^{-3})^{-2} / (6a^2b^{-1}c^{-2})^2$

19. Find the area of a triangle when a = 3/2 cm, b = 5/2 cm, and c = 2 cm. (see Heron's formula)

20. Explain the pattern of Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233....

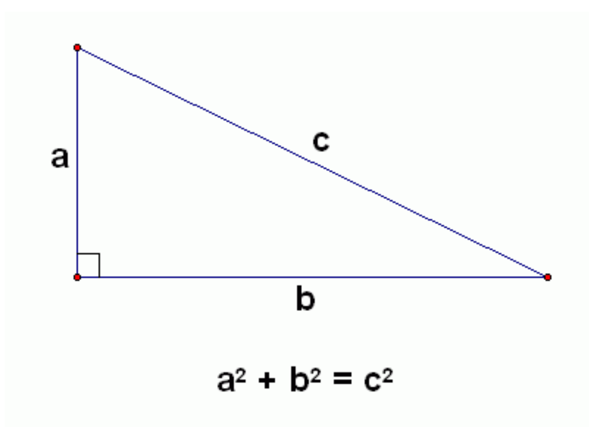
21. EC: Tell one math joke OR what is the most interesting insight/thought/mathematical tidbit that you have learned so far in this course?

FORMULAS & RELEVANT INFORMATION

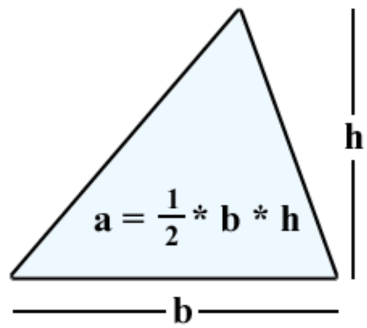
ROMAN NUMERALS

character	numerical value
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

PYTHAGOREAN THEOREM



AREA OF TRIANGLE FORMULA



Quadratic Formula:

If $ax^2 + bx + c = 0$ where $a \neq 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HERON'S FORMULA

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

with $s = \dfrac{a + b + c}{2}$

2 = 1
THE PROOF

1. Let *a* and *b* be equal non-zero quantities

   $$a = b$$

2. Multiply through by *a*

   $$a^2 = ab$$

3. Subtract b^2

   $$a^2 - b^2 = ab - b^2$$

4. Factor both sides

   $$(a - b)(a + b) = b(a - b)$$

5. Divide out $(a - b)$

   $$a + b = b$$

6. Observing that $a = b$

   $$b + b = b$$

7. Combine like terms on the left

   $$2b = b$$

8. Divide by the non-zero *b*

   $$2 = 1$$

MCCC

MAT 108

May 7, 2013

FINAL: TIPS FOR SUCCESS

In order to succeed on the final (May 9th, no makeups!), you should be confident with the following topics, concepts and ideas. Any questions you may have are your responsibility to seek clarification on, so please ask! (jzilahy@mc3.edu). The PDF files of the smartnotes will be your best source for preparation. The following topics are an overview and may or may not appear on the actual final, and any content that we covered in class is fair game.

You should be able to know and use the following:

Quadratic Formula

Rate x Time = Distance

Heron's Formula

Rules of Exponents

Basic probability (Venn Diagrams, cards & dice)

Pythagorean Theorem

Prime numbers (definition, factorization, and the Sieve of Erasthosthenes)

Add, subtract, multiply and divide fractions

Series & Sequences

Law of Cosines

Imaginary Numbers

Simple Ciphers

Cartesian Plane

Pascals Triangle and Combinations

Difference quotient

Basics of Binary

All germane historical events and people

MCCC

May 9th, 2013

<u>MAT108: FINAL</u>

each problem is worth 4 points

1. What would 42 and 100 be in binary?

2. Simplify the expression 1/x + 2/y + 3/z.

3. What are the first 3 prime numbers <u>after</u> 101?

4. Prime factorize 215

5. Given a right triangle with a side of length 42 and a hypotenuse of length 58, what is the length of the missing side?

6. What is the probability of getting an ace, red king, OR the queen of diamonds when selecting a single card out of a regular deck of 52 cards?

7. Two cars started from the same point, at 5 am, traveling in opposite directions at 40 and 50 mph respectively. <u>At what time</u> will they be 450 miles apart?

8. What is the common ratio in the following sequence: -500, 100, -20, 4,…?

9. If -500 is the first term in the sequence in the last problem find the series of the first 5 terms. (HINT: Series is a sum)

10. Simplify $\sqrt{-225}$.

11. In a football game, the quarterback throws a pass from the 12 yard line, 14 yards from the sideline. The pass is caught on the 51yard line, 6 yards from the same sideline. How long was the pass? (HINT: Draw a diagram first)

12. What is the one word that best describes what Calculus is about?

13. Use the Quadratic Formula to find the solutions to $x^2 + 2x - 8 = 0$.

14. Simplify the following exponent problem: $4(3x^2)^3$.

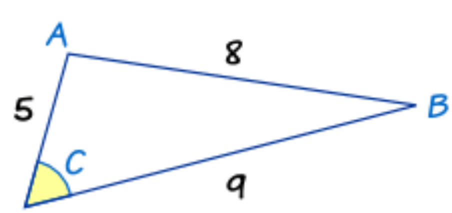
15. If $x = -3$ and $y = 7$, find the value of x^3y^3.

16. Find the sum, product, difference and quotient of 2/3 and 4/7.

17. Use the Difference Quotient to determine the derivative of the function $f(x) = 4x^2 + 2x$.

18. Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes, how many students are in neither class? How many are in either class? (HINT: Make a Venn Diagram)

19. What is angle C? (HINT: Use the Law of Cosines)



20. Which famous mathematician/philosopher is the Cartesian Plane named after?

21. How would you express $3^5 = 243$ logarithmically?

22. What is the name of the surface that has only one side?

23. Using the provided "Zebra" cipher, translate the following:

    IFSA FP ETQ Z ROAZJ

24. List at least 3 distinct ways to express slope.

25. If the top row in Pascal's triangle corresponds to row 0, then what is the sum of the 9^{th} row? (HINT: See attached Pascal's Triangle)

EC: Say at least one nice thing about your experience in this course.

FORMULAS & RELEVANT INFORMATION

PYTHAGOREAN THEOREM: $a^2 + b^2 = c^2$

LAW OF COSINES: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

DISTANCE FORMULA:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Plaintext alphabet: ABCDEFGHIJKLMNOPQRSTUVWXYZ

Ciphertext alphabet: ZEBRASCDFGHIJKLMNOPQTUVWXY

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

MCCC
MAT108
Mr. Zilahy
Spring 2013

Wolfram Alpha: Exploratory Lesson

Please note that while you can find many of these answers elsewhere, this project <u>*requires*</u> *all answers to originate from the website Wolfram Alpha. This project will be due by midnight Sunday (May 5th) to receive full credit.*

<u>*108 Queries:*</u>

1. Where was the Ishango bone discovered, and it is dated to how many years ago?

2. What is 444 in Roman numerals?

3. What is 444 in the Greek antique number system?

4. What is the basic rule from the Rhind Papyrus that was first published in 2002?

5. What is the decimal approximation of 1/3 + 2/7 + 7/9 + 11/23 to 17 digits of accuracy?

6. What is the 42nd prime number?

7. What is the best known Pythagorean Triple?

8. How does WA interpret $(x^{-3})^2/x^{-7}$ as input? What is the answer?

9. Using the Law of Cosines, find the angle measures (in degrees) of a triangle with side lengths of 12, 17, 19.

10. What is i^{18}?

11. What percent of the world population speaks French?

12. What is the decimal approximation of the natural logarithm of 12 to 5 digits of accuracy?

13. What is the word "Leet" in leetspeak (cipher)?

14. What is 1337 as a phrase? From what word does it derive?

15. What does WA list Rene Descartes profession as?

16. What is the distance (decimal approximation) between the origin and the ordered pair (45,45)?

17. Which trigonometric function is the definition of “slope” defined as?

18. What is the basic definition of Pascal’s Triangle?

19. What is the basic definition of Calculus?

20. What type of diagram is the “seed of life”? Recreate for an extra credit point!

Open Ended Queries:

21. Calculate the precise distance in miles between Philadelphia and New York.

22. Ask Wolfram Alpha "where am i"

23. How long would it take to read, speak and type 300 pages of a typical book?

24. Compute the flight time from Los Angeles to Philadelphia.

25. What number is the "Answer to Life, the Universe, and Everything"?

26. What is the first comparison for 88 mile per hour?

27. Are we there yet?

28. How old is Wolfram Alpha? (in hours)

29. How do you win the lottery?

30. How many licks does it take to get to the center of a tootsie pop?

EC: Discover and report one really cool thing that Wolfram Alpha can compute.

EC2: Run your own personal analysis on your facebook at:

http://www.wolframalpha.com/facebook/ and report any interesting facts.

FIN